1.6. Dimensional Analysis:

Dimensional analysis is a method of calculation utilizing a knowledge of units. It is an easy way to convert from one unit of measure to another by multiplying be an appropriate conversion factor.

Conversion factors are used to manipulate units. It a fraction in which numerator and denominator are in different units, but equal to the same quantity. The algebric value of the converstion factor is always 1.

If a = b, then a/b = 1 and b/a = 1

1 ft = 12 in so, 1 ft/12 in = 1 and 12 in/1ft = 1

Given units can be multiplied and divided to give the desired units.

given unit x (conversion factor) = Desired unit

conversion factor = (desired unit) / (given unit).

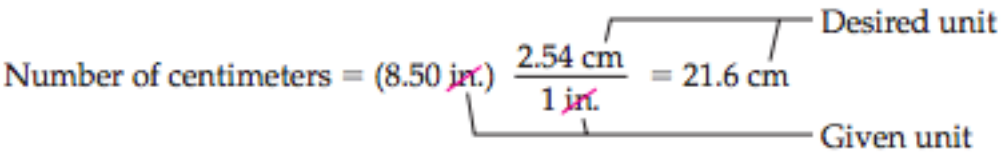
$$\cancel{\text{Given unit}} \times \frac{\text{desired unit}}{\cancel{\text{given unit}}} = \text{desired unit}$$

We often need to use more than one conversion factor in order to complete a problem. The final answer must be the correct unit.

For example, 2.54 cm and 1 in. are the same length, in. This relationship allows us to write two conversion factors: 2.54cm 1 in. and 1 in. 2.54cm.

$$\frac{2.54\,\text{cm}}{1\,\text{in.}} \quad \text{and} \quad \frac{1\,\text{in.}}{2.54\,\text{cm}}$$

What is the length in centimeters of an object that is 8.50 in. long?



Using two or more conversion factors:

For example: Let us convert the length of an 8.00-m rod to inches.
We know that: 1 cm = 10^{-2} m, and 1 in = 2.54 cm.

$$\text{Number of inches} = (8.00\ \text{m})\left(\frac{1\ \text{cm}}{10^{-2}\ \text{m}}\right)\left(\frac{1\ \text{in.}}{2.54\ \text{cm}}\right) = 315\ \text{in.}$$

Conversions Involving Volume:

Suppose that we wish to know the mass in grams of 2.00 cubic inches of gold given that the density of the gold is 19.3 g/cm^3

We know the following conversion factors:

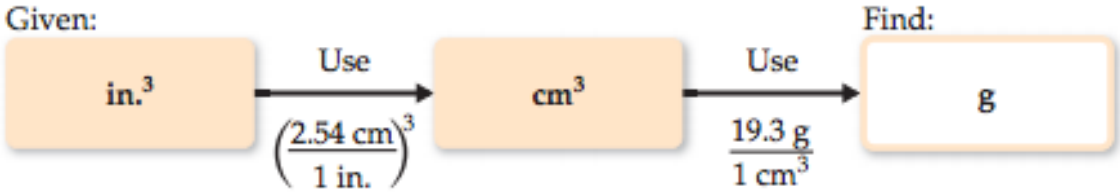
1 cm^3 = 19.3 g and 2.54 cm = I inch

We can write the following conversion factors:

$$\frac{19.3\ \text{g}}{1\ \text{cm}^3} \quad \text{and} \quad \frac{1\ \text{cm}^3}{19.3\ \text{g}}$$

and

$$\frac{(2.54\text{ cm})^3}{(1\text{ in.})^3} = \frac{(2.54)^3\text{ cm}^3}{(1)^3\text{ in.}^3} = \frac{16.39\text{ cm}^3}{1\text{ in.}^3}$$



$(2.00\text{ in.}^3)(2.54\text{ cm/in.})^3\ (19.3\text{ g gold} / 1\text{ cm}^3) = 6.33\text{ g gold}$

$$\text{Mass in grams} = (2.00\ \cancel{\text{in.}^3})\left(\frac{16.39\ \cancel{\text{cm}^3}}{1\ \cancel{\text{in.}^3}}\right)\left(\frac{19.3\text{ g}}{1\ \cancel{\text{cm}^3}}\right) = 633\text{ g}$$

Summary of Dimensional Analysis:

Three questions:

1. What data are we given?
2. What quantity do we need?
3. What conversion factors are available to take us from what we are given to what we need?