Graphical Velocity and Acceleration

Questions?

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P2.1D Describe and analyze the motion that a position-time graph represents, given the graph.

P2.2C Describe and analyze the motion that a velocity-time graph represents, given the graph.

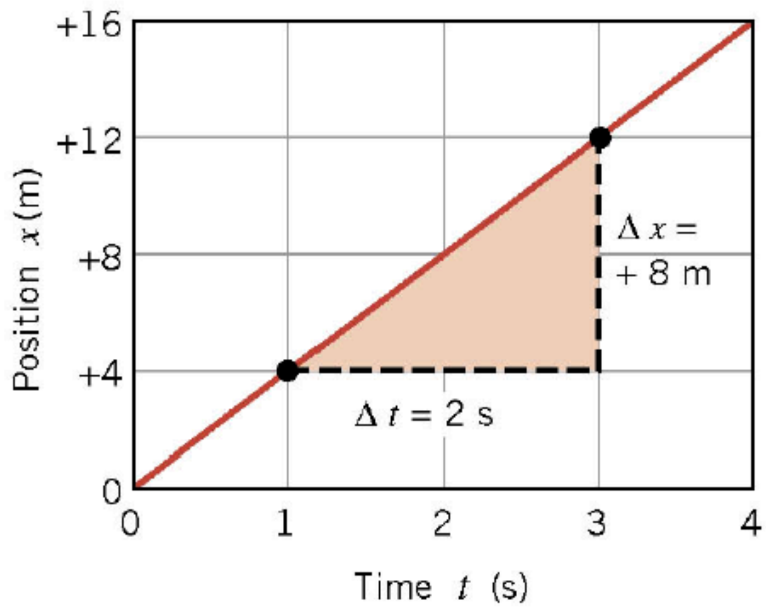
P2.2e Use the area under a velocity-time graph to calculate the distance traveled and the slope to calculate the acceleration.

Position versus Time Graph and Constant Velocity

Suppose a bicyclist is riding with a constant velocity of v = 4 m/s. The position x of the bicycle can be plotted along the vertical axis of a graph. The time t is plotted along the horizontal axis. A graph of position versus time is shown below.

Since the position of the bike increases by 4 m every second, the graph of x versus t is a straight line. Furthermore, if the bike is assumed to be at x = 0 m when t = 0 s, the straight line passes through the origin. Each point on this line gives the position of the bike at a particular time. For instance, at t 1 s the position is 4 m, while at t 3 s the position is 12 m.

The velocity could be determined by considering what happens to the bike between the times of 1 and 3 s, for instance. The change in time is Δt = (3-1) = 2 s. During this time interval, the position of the bike changes from **+** 4 to **+**12 m, and the change in position is x = (12-4) = 8 m. The ratio Δx /Δt is called the slope of the straight line.

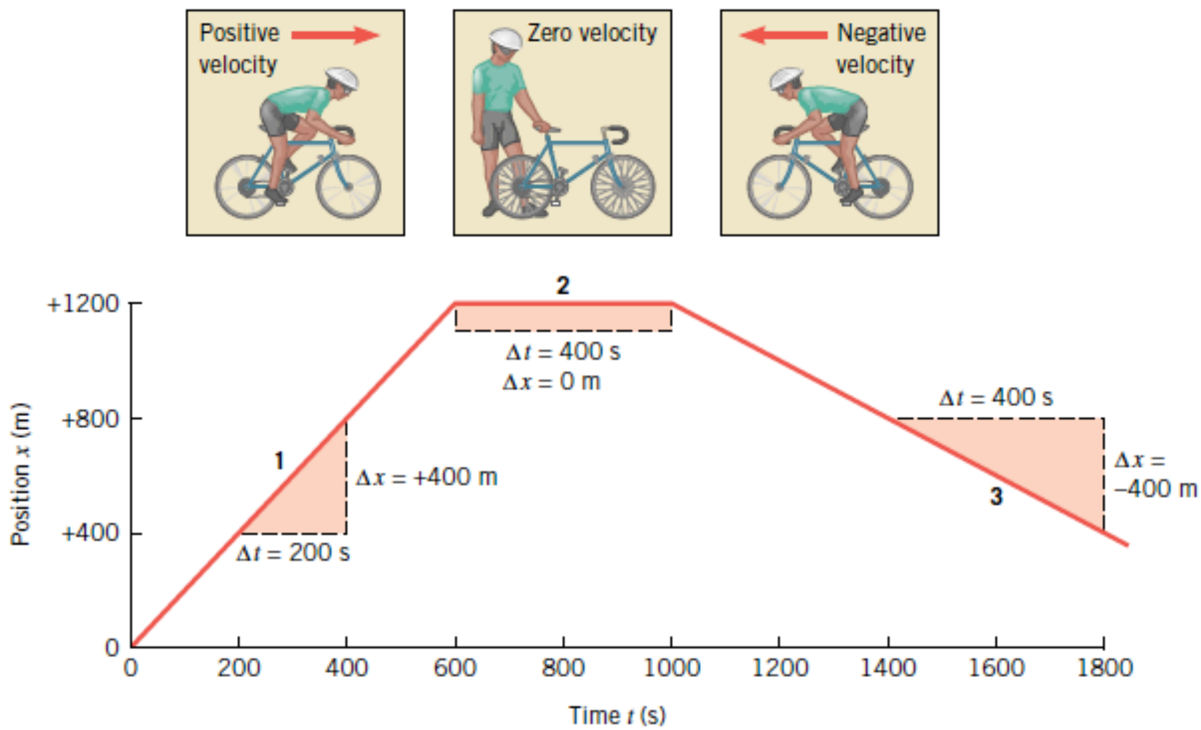


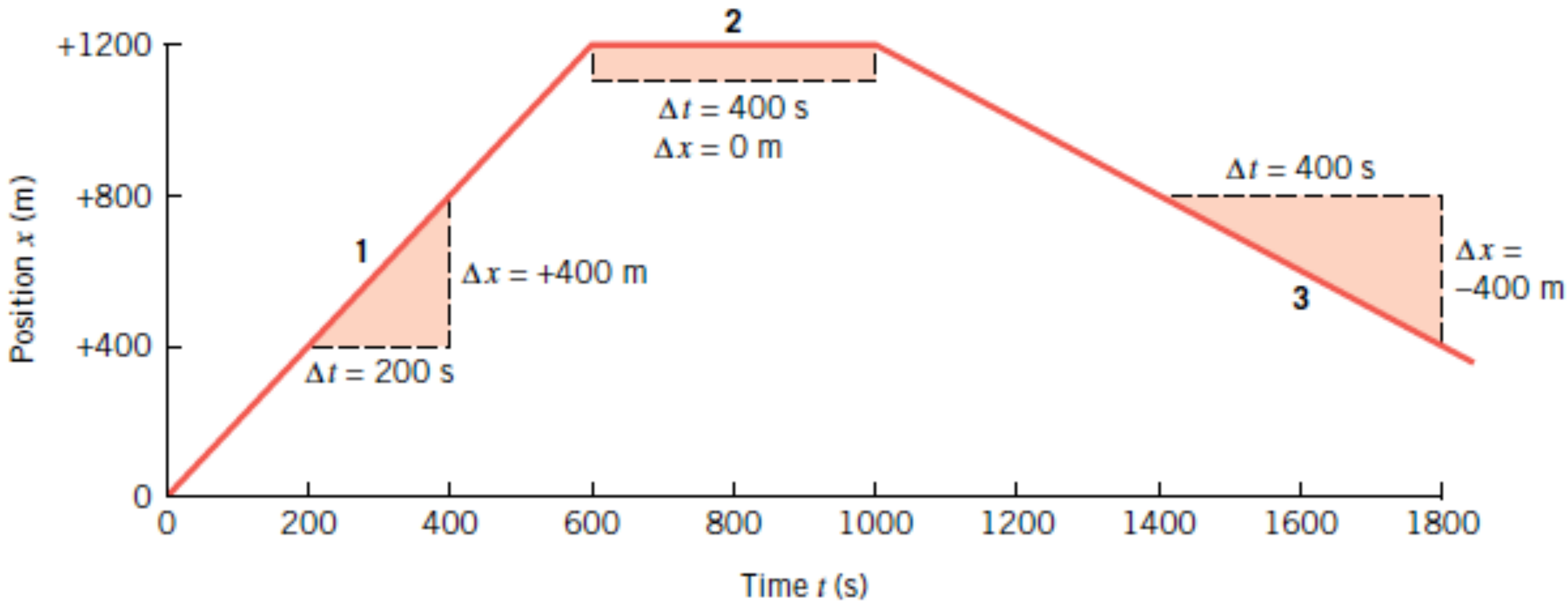
$$\text{Slope} = \frac{\Delta x}{\Delta t} = \frac{+8\,\text{m}}{2\,\text{s}} = +4\,\text{m/s} \; = v$$

Thus, for an object moving with a constant velocity, the slope of the straight line in a position– time graph gives the velocity.

Since the position– time graph is a straight line, any time interval t can be chosen to calculate the velocity. Choosing a different t will yield a different x ,but the velocity **Δ**x /**Δ**t will not change.

In the real world, objects rarely move with a constant velocity at all times, as illustrated in the position versus time graph of a bicycle trip shown below. A bicyclist maintains a constant velocity on the outgoing leg of a trip (segment 1), zero velocity while stopped (segment 2) , and another constant velocity on the way back (segment 3). Using the time and position intervals indicated in the drawing, obtain the velocities for each segment of the trip.





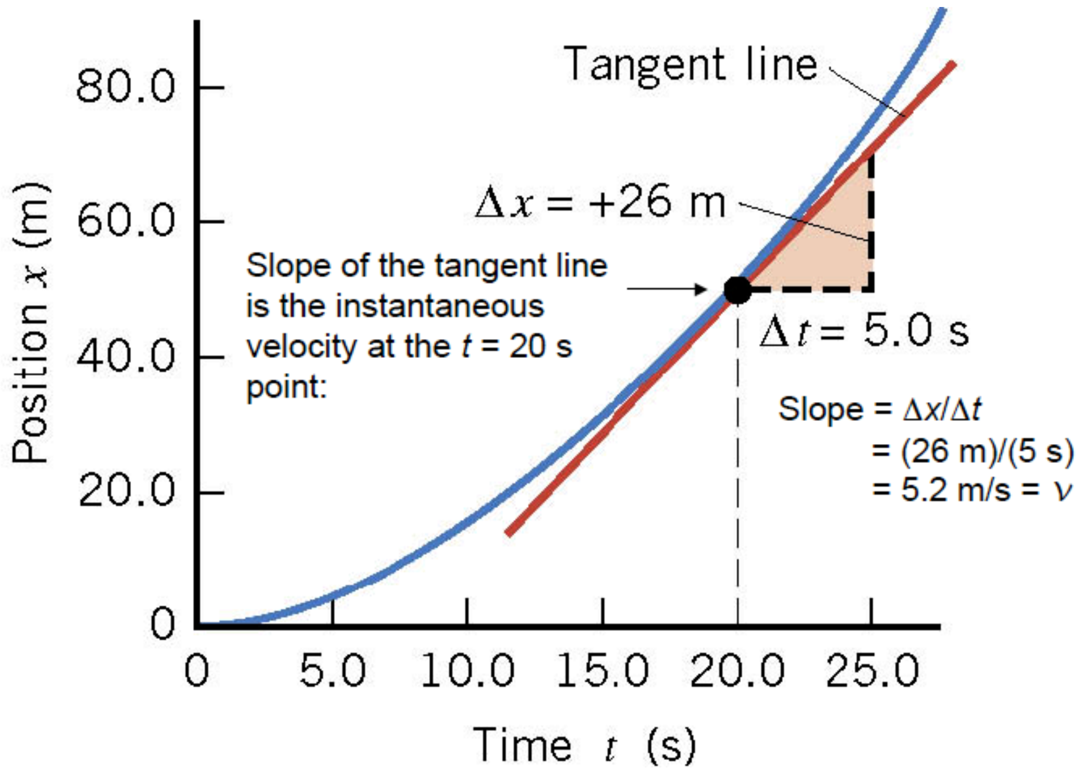
Solution The average velocities for the three segments are

Segment 1 $$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{800\text{ m} - 400\text{ m}}{400\text{ s} - 200\text{ s}} = \frac{+400\text{ m}}{200\text{ s}} = \boxed{+2\text{ m/s}}$$

Segment 2 $$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{1200\text{ m} - 1200\text{ m}}{1000\text{ s} - 600\text{ s}} = \frac{0\text{ m}}{400\text{ s}} = \boxed{0\text{ m/s}}$$

Segment 3 $$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{400\text{ m} - 800\text{ m}}{1800\text{ s} - 1400\text{ s}} = \frac{-400\text{ m}}{400\text{ s}} = \boxed{-1\text{ m/s}}$$

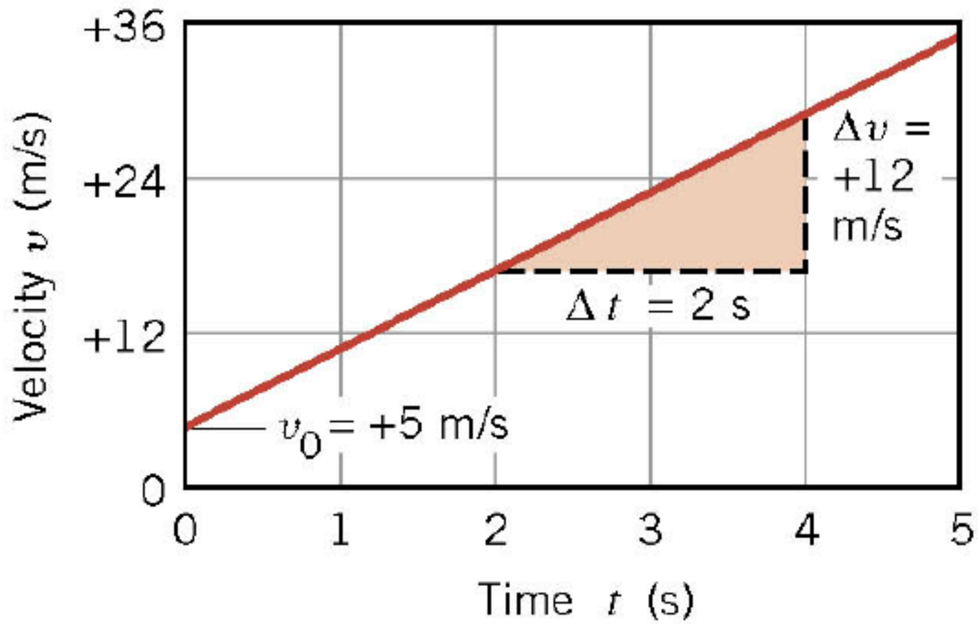
Object Moving with Changing Velocity



When the velocity is changing, the position-vs.-time graph is a curved line. The slope **Δx/Δt** of the tangent line drawn to the curve at a given time is the instantaneous velocity at that time.

Velocity versus Time Graph and Acceleration

The slope of a plot of velocity versus time gives the object's acceleration.Object moving with constant acceleration.

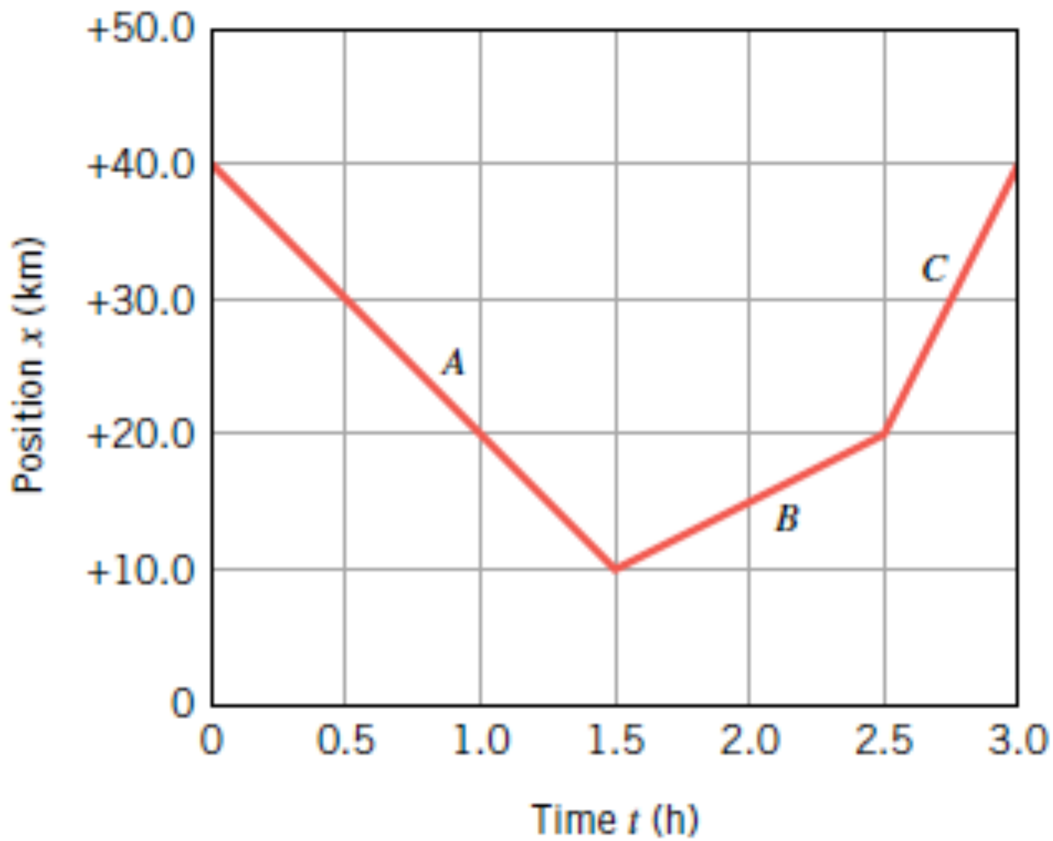


$$\text{Slope} = \frac{\Delta v}{\Delta t} = \frac{+12\ \text{m/s}}{2\ \text{s}} = +6\,\text{m/s}^2 = a$$

A velocity-vs.-time graph that applies to an object with an acceleration of $\Delta \mathbf{v}/\Delta t = 6\ \text{m/s}^2$. The initial velocity is $\mathbf{v}_0 = 5$ m/s when $t = 0$ s.

Practice Problems: Answer questions 1 and 2.

1. A bus makes a trip according to the position–time graph shown in the drawing. What is the average velocity (magnitude and direction) of the bus during each of the segments labeled **A**, **B**, and **C**? Express your answers in km /h.



2. A person who walks for exercise produces the position–time graph shown below. (a) Without doing any calculations, decide which segments of the graph (**A**, **B**, **C**) indicate positive,negative, and zero average velocities. (b) Calculate the average velocity for each segment to verify your answers to part (a).