Conservation of Momentum - Class Notes

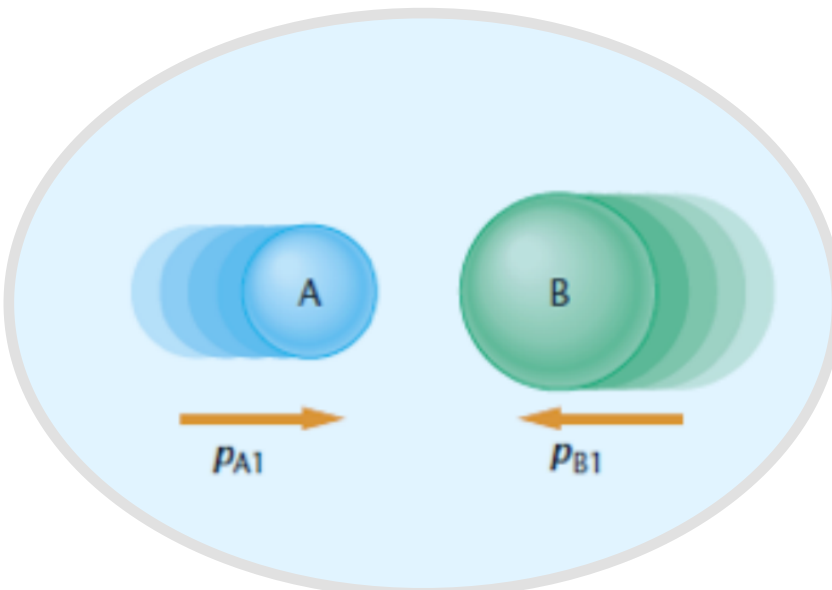
Western International High School

Nada Saab, Ph.D

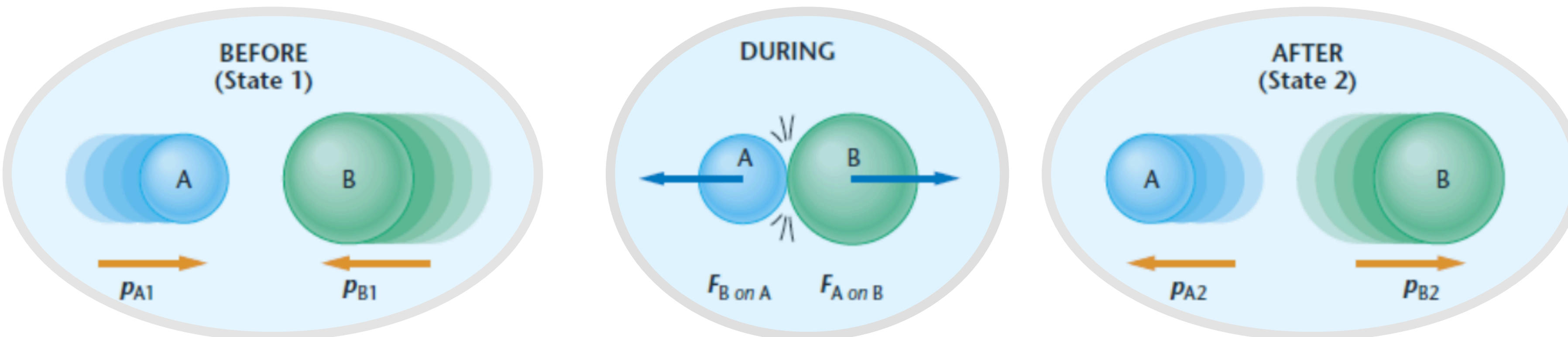
2020-2021

Example 1: Isolated system : Isolated, no external force is acting on it. It is fine if things changed inside. Both balls have linear momentum:

P = m x V



Conservation of linear momentum (P = m x V):
Total Linear Momentum (P_{total}) stays the same (<u>does not change</u>).



(P_{total}) state 1 = (P_{total}) = (P_{total}) state 2

$P_{A1} + P_{B1} = P_{A2} + P_{B2}$

$(m_A \times V_{A1}) + (m_B \times V_{B1}) = (m_A \times V_{A2}) + (m_B \times V_{B2})$

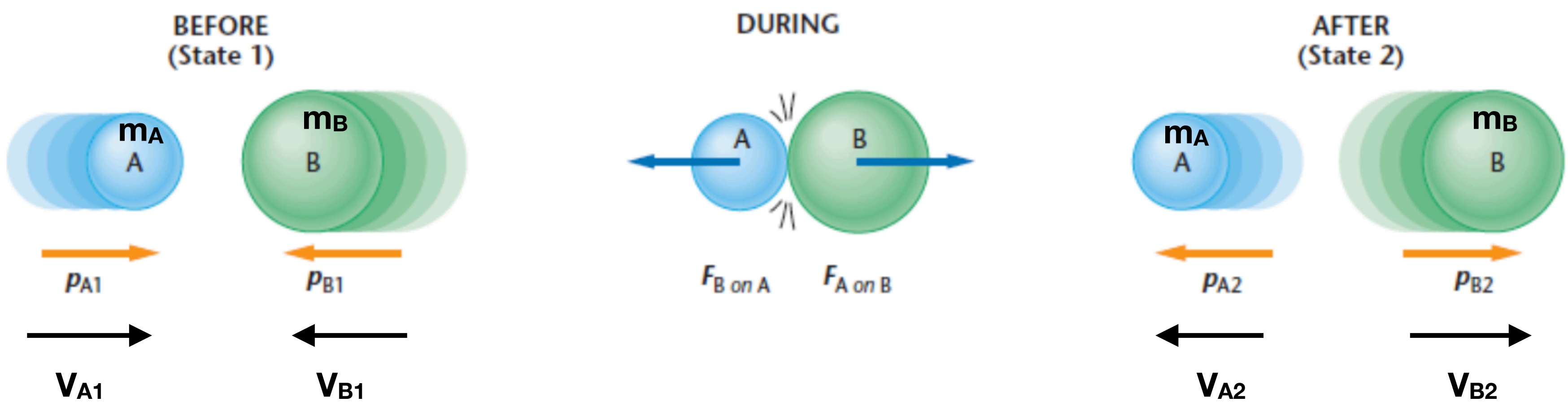
Total momentum before collision = Total momentum after collision
(State 1) (State 2)

(P_{total}) before collision = (P_{total}) after collision

$$P_{A1} + P_{B1} = P_{A2} + P_{B2}$$

$$(m_A \times V_{A1}) + (m_B \times V_{B1}) = (m_A \times V_{A2}) + (m_B \times V_{B2})$$

Sum of all (m x v) (before the collision) = Sum of all (m x v) (after the collision)



Total momentum does not change.

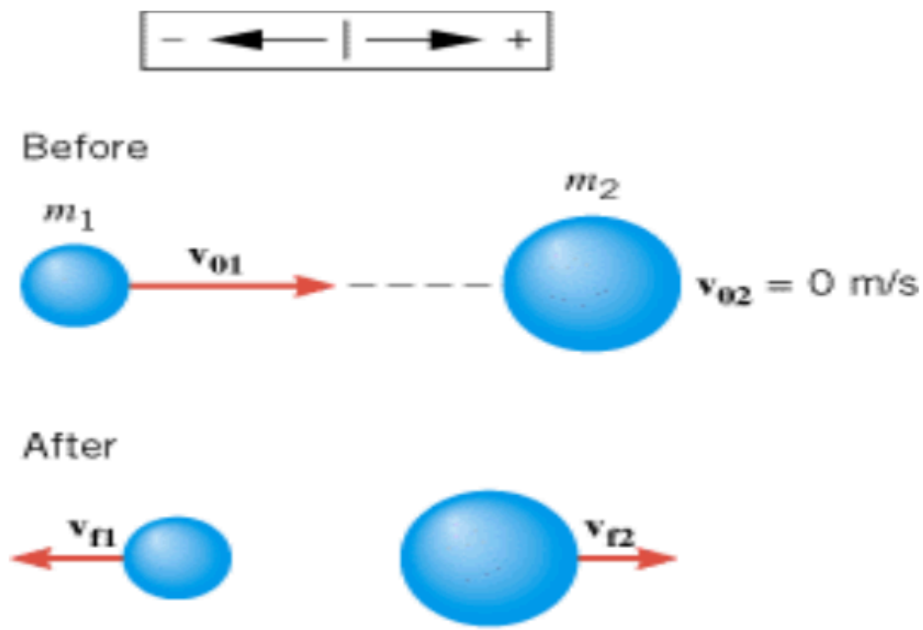
Momentum and velocity have the same direction.

Mass does not change.

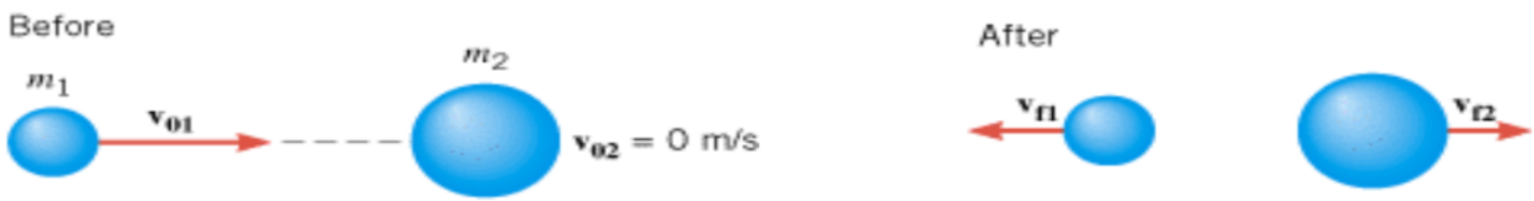
Velocity changes from before the collision (state 1) to after the collision (state 2).

Example 2: Before Collision: A softball of mass m1 (0.200 kg) moving at a constant velocity v01 (8.3 m/s) strikes a second ball of mass m2 (0.45 kg) that is at rest (v02 = 0 m/s).

After Collision, the incoming ball rebounds straight back at a velocity vf1 (3.2 m/s) and the second ball moves at a velocity vf2. Calculate vf2 (final velocity)



Data Table					
m_2	m_1	v_{02}	v_{01}	v_{f1}	v_{f2}
0.45 kg	0.200 kg	0 m/s	8.3 m/s	-3.2 m/s	?



Total momentum before collision = Total momentum after collision
(balls 1 and 2) (balls 1 and 2)

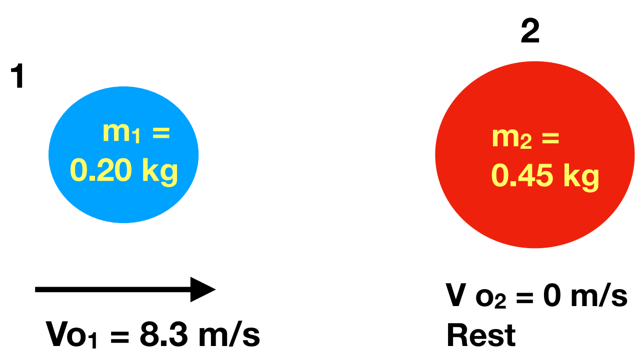
(P_{total}) before collision = (P_{total}) after collision

$$P_{o1} + P_{o2} = P_{f1} + P_{f2}$$

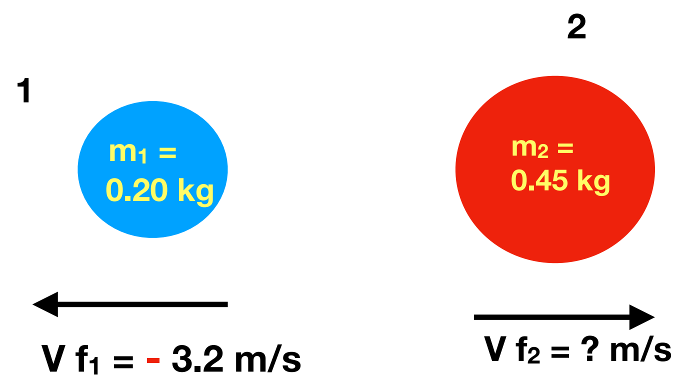
$$(m_1 \times V_{o1}) + (m_2 \times V_{o2}) = (m_1 \times V_{f1}) + (m_2 \times V_{f2})$$

Before Collision: A softball of mass m1 (0.200 kg) moving at a constant velocity V01 (8.3 m/s) strikes a second ball of mass m2 (0.45 kg) that is at rest (v02 = 0 m/s). **After Collision,** the incoming ball rebounds straight back at a velocity vf1 (3.2 m/s) and the second ball moves at a velocity vf2. Calculate vf2 (final velocity)

Before Collision



After the Collision



Data Table					
m_2	m_1	v_{02}	v_{01}	v_{f1}	v_{f2}
0.45 kg	0.200 kg	0 m/s	8.3 m/s	-3.2 m/s	?

Before Collision (o)

1

$m_1 =$ 0.20 kg

$m_2 =$ 0.45 kg

$V\ o_1 = 8.3$ m/s

$V\ o_2 = 0$ m/s, Rest

momentum of ball 1

$P_{o1} = m_1 \times V_{o1}$

$P_{o2} = m_2 \times V_{o2}$

After the Collision (f)

$m_1 =$ 0.20 kg

$m_2 =$ 0.45 kg

$V\ f_1 = -\ 3.2$ m/s

$V\ f_2 = ?$ m/s

momentum of ball 1

$P_{f1} = m_1 \times V_{f1}$

momentum of ball 2

$P_{f2} = m_2 \times V_{f2}$

Total momentum before collision = Total momentum after collision

$$P_{o1} + P_{o2} = P_{f1} + P_{f2}$$

$$(m_1 \times V_{o1}) + (m_2 \times V_{o2}) = (m_1 \times V_{f1}) + (m_2 \times V_{f2})$$

Data Table					
m_2	m_1	v_{02}	v_{01}	v_{f1}	v_{f2}
0.45 kg	0.200 kg	0 m/s	8.3 m/s	-3.2 m/s	?

Total momentum before collision = Total momentum after collision
(balls 1 and 2) (balls 1 and 2)

(P_{total}) before collision = (P_{total}) after collision

$$P_{o1} + P_{o2} = P_{f1} + P_{f2}$$

$$(m_1 \times V_{o1}) + (m_2 \times V_{o2}) = (m_1 \times V_{f1}) + (m_2 \times V_{f2})$$

$$(0.200 \times 8.3) + (0.45 \times 0) = (0.200 \times -3.2) + (0.45 \times \mathbf{V_{f2}})$$

$$(1.66) + 0 = -0.64 + (0.45 \times \mathbf{V_{f2}})$$

$$(1.66) + 0 + 0.64 = - 0.64 + 0.64 + (0.45 \times \mathbf{V_{f2}})$$

$$2.3 = (0.45 \times V_{f2})$$

$$\frac{2.3}{0.45} = \frac{0.45}{0.45} \times V_{f2}$$

$$5.1 \text{ m/s} = V_{f2}$$

Homework (2):

Before Collision: A 0.1 kg ball (**blue**) moving at a constant velocity of 0.2 m/s strikes a 0.4 kg ball (**red)** that is at rest.

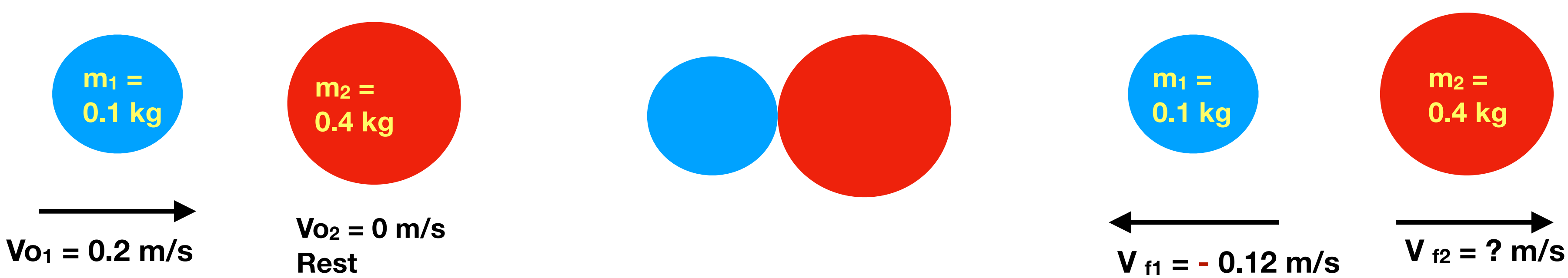
After the collision, the first ball (**blue**) rebounds straight back at 0.12 m/s.

Calculate the final velocity of the second ball (**red)**.

Data Table					
m_1	m_2	V_{01}	V_{02}	V_{f1}	V_{f2}
0.1 kg	0.4 kg	0.2 m/s	0 m/s	- 0.12 m/s	?

Before Collision (o)

After Collision (f)



Before Collision (o)

$m_1 =$ 0.1 kg

$m_2 =$ 0.2 kg

$V\ o_1 = 0.2$ m/s

$V\ o_2 = 0$ m/s, Rest

After the Collision (f)

$m_1 =$ 0.1 kg

$m_2 =$ 0.2 kg

$V\ f_1 = -\ 1.2$ m/s

$V\ f_2 =$? m/s

momentum of ball 1 P_{o1} + **momentum of ball 2 P_{o2}** = **momentum of ball 1 P_{f1}** + **momentum of ball 2 P_{f2}**

$$m_1 \times V_{o1} + m_2 \times V_{o2} = m_1 \times V_{f1} + m_2 \times V_{f2}$$

Data Table					
m_1	**m_2**	**V_{01}**	**V_{02}**	**V_{f1}**	**V_{f2}**
0.1 kg	0.4 kg	0.2 m/s	0 m/s	- 0.12 m/s	?

Collision = Conservation of momentum

Total momentum before collision = Total momentum after collision
(balls 1 and 2) (balls 1 and 2)

$$P_{o1} + P_{o2} = P_{f1} + P_{f2}$$

$$(m_1 \times V_{o1}) + (m_2 \times V_{o2}) = (m_1 \times V_{f1}) + (m_2 \times V_{f2})$$

$$(0.1 \times 0.2) + (0.4 \times 0) = (0.1 \times -0.12) + (0.4 \times V_{f2})$$

$$(0.02) + (0) = (-0.012) + (0.4 \times V_{f2})$$

$$0.02 = -0.012 + (0.4 \times V_{f2})$$

$$0.02 + 0.012 = -\,0.012 + 0.012 + (\,0.4 \times V_{f2})$$

$$0.02 + 0.012 = (\,0.4 \times V_{f2})$$

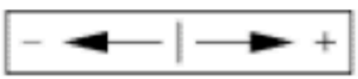
$$0.032 = 0.4 \times V_{f2}$$

$$\frac{0.032}{0.4} = \frac{0.4}{0.4} \times V_{f2}$$

$$0.08 \text{ m/s [Forward]} = V_{f2}$$

Example 3: A loaded railway car of mass 6000 kg is rolling to the right at 3.0 m/s. It collides and couples with an empty freight car of mass 3000 kg, rolling to the right on the same track at 2.0 m/s . What is the speed and direction of the pair after the collision?

Data Table				
m_1	m_2	V_{01}	V_{02}	$V_{f1} = V_{f2} = V_f$
3000 kg	6000 kg	2.0 m/s	3.0 m/s	?



(a) Before (b) After

Collision = Conservation of momentum

Total momentum before collision = Total momentum after collision

$$P_{o1} + P_{o2} = P_{f1} + P_{f2}$$

$$(m_1 \times V_{o1}) + (m_2 \times V_{o2}) = (m_1 \times V_f) + (m_2 \times V_f)$$

$$(m_1 \times V_{o1}) + (m_2 \times V_{o2}) = (m_1 + m_2) \times V_f$$

Data Table				
m_1	**m_2**	**V_{01}**	**V_{02}**	**$V_{f1} = V_{f2} = V_f$**
3000 kg	6000 kg	2.0 m/s	3.0 m/s	?

$$(m_1 \times V_{o1}) + (m_2 \times V_{o2}) = (m_1 + m_2) \times V_f$$

$$(3000 \times 2.0) + (6000 \times 3.0) = (3000 + 6000)\, V_f$$

$$6000 \quad + \quad 18000 \quad = \quad 9000\ V_f$$

$$6000 \quad + \quad 18000 \quad = \quad 9000\ V_f$$

$$24000 = 9000\ V_f$$

$$\frac{24000}{9000} = \frac{9000\ V_f}{9000}$$

2.66 m/s [Forwards] = V_f

Example 4 : (a)A loaded railway car of mass 6000 kg is rolling to the right at 2.0 m/s. Another empty freight car of mass 3000 kg is rolling to the left on the same track at 3.0 m/s. (b)They collide and couple. What is the speed and direction of the pair after the collision?

Data Table				
m_1	m_2	V_{01}	V_{02}	$V_{f1} = V_{f2} = V_f$
3000 kg	6000 kg	- 3.0 m/s (left)	2.0 m/s	?

(a) Before (b) After

Data Table				
m_1	**m_2**	**V_{01}**	**V_{02}**	**$V_{f1} = V_{f2} = V_f$**
3000 kg	6000 kg	- 3.0 m/s	2 m/s	?

$$(m_1 \times V_{o1}) + (m_2 \times V_{o2}) = (m_1 + m_2) \times V_f$$

$$(m_1 \times V_{o1}) + (m_2 \times V_{o2}) = (m_1 \times V_f) + (m_2 \times V_f)$$

$$(3000 \times -3.0) + (6000 \times 2) = (3000 \times V_f) + (6000 \times V_f)$$

$$(3000 \times -3.0) + (6000 \times 2) = (3000 + 6000) \times V_f$$

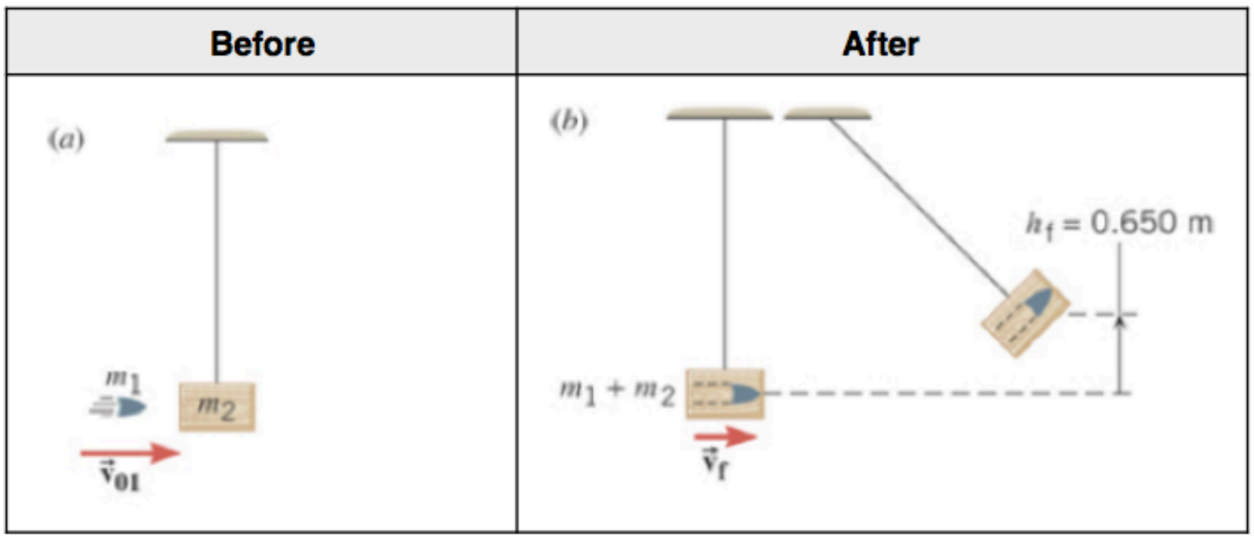
$$(-9000) + (12000) = (9000) \times V_f$$

$$3000 = 9000 \times V_f$$

$$\frac{3000}{9000} = \frac{9000}{9000} \times V_f$$

$$0.33 \text{ m/s [Forwards]} = V_f$$

Example 5: A bullet of mass (m1 = 0.0100 kg) is fired with a speed of (v01 = 896 m/s) and collide with a block of wood of mass (m2 = 2.5 kg) at rest (v02 = 0 m/s) and suspended by a wire with negligible mass . After the collision (b), they both swing to a height of 0.0650 m. Find the final velocity (vf) of the wood and the bullet.



Data Table				
m_1	m_2	V_{01}	V_{02}	$V_{f1} = V_{f2} = V_f$
0.0100 kg	2.5 kg	896 m/s	0 m/s	?

Data Table				
m_1	**m_2**	**V_{01}**	**V_{02}**	**$V_{f1} = V_{f2} = V_f$**
0.0100 kg	2.5 kg	896 m/s	0 m/s	?

Total momentum before collision = Total momentum after collision

$$P_{o1} + P_{o2} = P_{f1} + P_{f2}$$

$$(m_1 \times V_{o1}) + (m_2 \times V_{o2}) = (m_1 \times V_f) + (m_2 \times V_f)$$

$$(m_1 \times V_{o1}) + (m_2 \times V_{o2}) = (m_1 + m_2) \times V_f$$

$$(0.0100 \times 896) + (2.5 \times 0) = (0.0100 \times V_f) + (2.5 \times V_f)$$

$$(0.0100 \times 896) + (2.5 \times 0) = (0.0100 + 2.5) \times V_f$$

$$8.96 + 0 = 2.51\ V_f$$

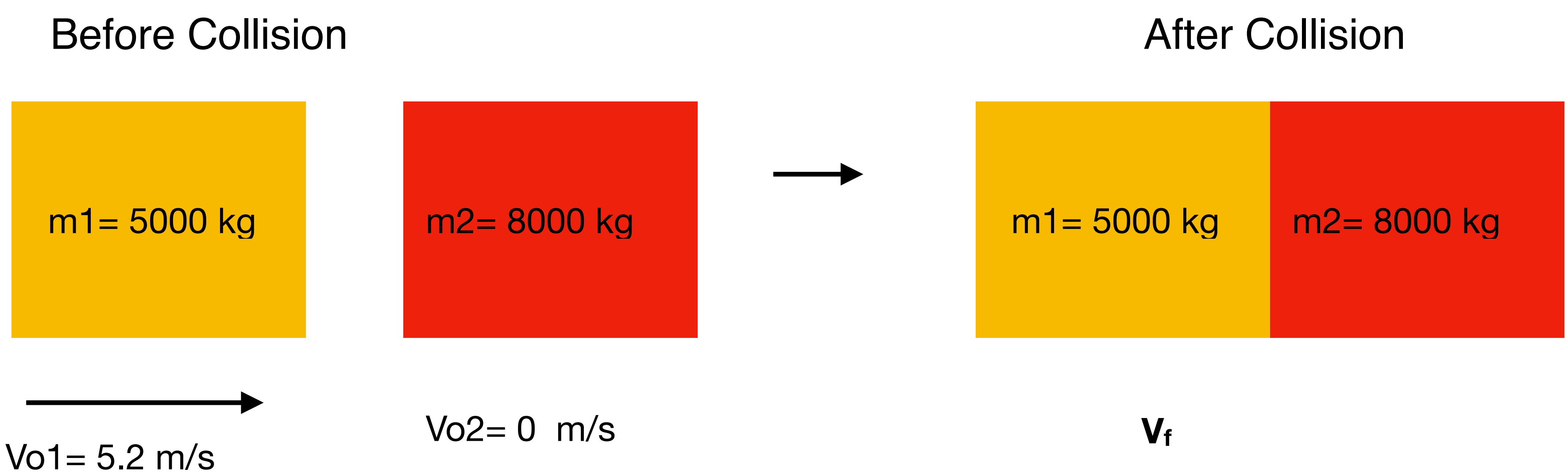
$$8.96 = 2.51 \times V_f$$

$$\frac{8.96}{2.51} = \frac{2.51}{2.51} V_f$$

$$3.57 \text{ m/s} = V_f$$

Homework (a) : A 5000 kg boxcar moving at 5.2 m/s on a level, frictionless track runs into a stationary (rest = not moving) 8000 kg tank car. If they hook together in the collision, how fast will they be moving afterwards?

Data Table				
m_1	m_2	V_{01}	V_{02}	$V_{f1} = V_{f2} = V_f$
5000 kg	8000 kg	5.2 m/s	0 m/s	?



Data Table				
m_1	**m_2**	**V_{01}**	**V_{02}**	**V_f**
5000 kg	8000 kg	5.2 m/s	0 m/s	?

Total momentum before collision = Total momentum after collision

$$(m_1 \times V_{o1}) + (m_2 \times V_{o2}) = (m_1 + m_2) \times V_f$$

$$(5000 \times 5.2) + (8000 \times 0) = (5000 + 8000) \times V_f$$

$$26000 + 0 = 13000 \times V_f$$

$$26000 = 13000 \times \mathbf{V}_f$$

$$\frac{26000}{13000} = \frac{13000}{13000} \times V_f$$

$$\mathbf{2\ m/\ s = V_f}$$