Conservation of Momentum

by

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P3.3b Predict how the change in velocity of a small mass compares to the change in velocity of a large mass when the objects interact (e.g., collide).

P3.5a Apply conservation of momentum to solve simple collision problems.

Items:

1- Conservation of Momentum

2- Collision of two objects moving in the same direction.

3- Collision of two objects moving in opposite directions.

4- Collision of two objects, one at rest and one is moving.

5- Two objects starting at rest and pushing each other.

PRINCIPLE OF CONSERVATION OF LINEAR MOMENTUM

The law of conservation of momentum is one of the most important laws in science. It can be used to explain what happens when objects collide.

If the net external force (F_{net}) acting on a system of objects is zero, then the total linear momentum (P_{total}) of the system remains unchanged (constant).

Conservation of linear momentum explains the movement of billiard balls after the collision of the scattering of bowling pins by a bowling ball.

Total momentum before the collision = Total momentum after the collision

$$\vec{\mathbf{p}}_{\mathbf{total}} \text{ (before the impact or collision)} = \vec{\mathbf{p}}_{\mathbf{total}} \text{ (after the impact or collision)}$$

The net external force (F_{net}) is zero when the system of objects is isolated. This means that the objects are not pulled or pushed by anything outside their system. It is okay if they push and pull each other within the system.

Conservation of Momentum
When $\vec{F}net = 0$ **then** $\vec{P}_{total}$ is constant
Total momentum before the collision = Total momentum after the collision
$\vec{\mathbf{p}}$ **total** (**before the collision**) = $\vec{\mathbf{p}}$ **total** (**after the collision**)
Recall: P is the momentum = m v,
Sum of all (m x v) (**before the collision**) = Sum of all (m x v) (**after the collision**)

Steps for solving problems using the law of conservation of momentum

In applying conservation of momentum:

a) Define the isolated system, where the net external force is zero.

b)Sketch two pictures: one for the system before the collision and one for the system after the collision.

c)Defines which direction is chosen as the positive direction, then the opposite direction would be the negative direction. The momentum has the same direction of the velocity.

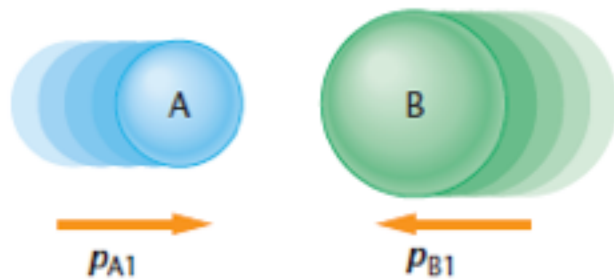
d)write the statement that the total momentum before the collision is the same as the total momentum after the collision.

Total momentum before the collision = Total momentum after the collision

e) Write it as an equation too. Solve the equation.

Example 1: *Conservation of momentum of billiard balls.*

Two billiard balls A and B are moving toward collision. The momentum of the ball A before the collision is P_{A1}, and the momentum of the ball B before the collision is P_{B1}.

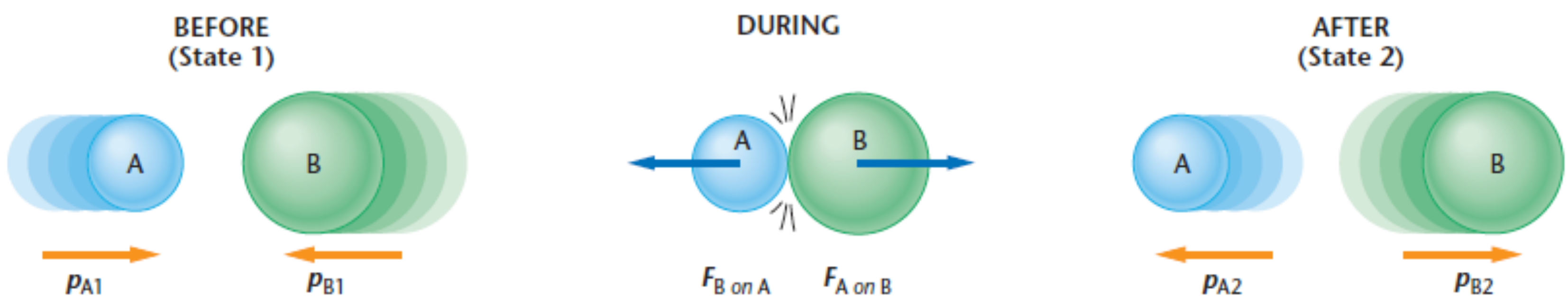


Compare the momentum of the balls before and after the collision.

Steps to solve the problem:

a) The net force acting on the two balls above is zero. So, the system is isolated.

b) This is a sketch of the collision with the two states: State 1 for before the collision and State 2 for after the collision.



c) The direction to the right is the positive direction. So, the direction to the left is negative. The momentum P has the same direction of the velocity.

Then, P_{A1} is positive and P_{B1} is negative.

Then, P_{A2} is negative and P_{B2} is positive.

d) When the two balls collide, the momentum (p) is conserved or remains the same.

Total momentum before the collision = Total momentum after the collision

e) Equation:

$$P_{A1} + P_{B1} = P_{A2} + P_{B2}$$

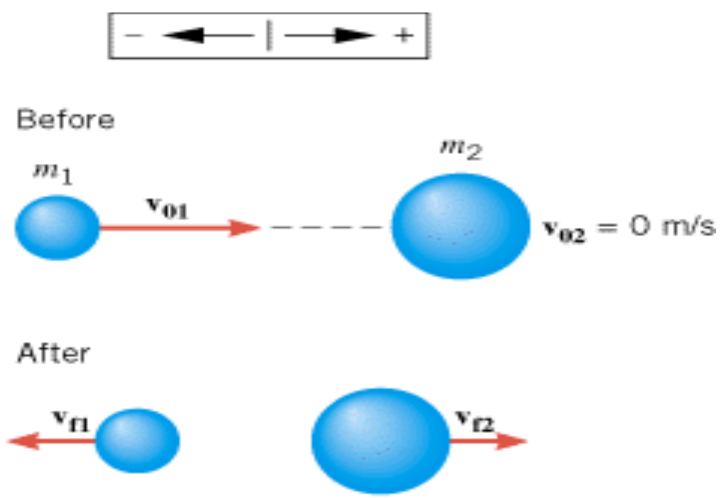
Sum of all (m x v) (**before the collision**) = Sum of all (m x v) (**after the collision**)

$$(m_{A1} \times v_{A1}) + (m_{B1} \times v_{B1}) = (m_{A2} \times v_{A2}) + (m_{B2} \times v_{B2})$$

Example 2: *A Ball Striking Another Ball at Rest.*

Before : A softball of mass m_1 (0.200 kg) moving at a constant velocity v_{01} (8.3 m/s) strikes a second ball of mass m_2 (0.45 kg) that is at rest (v_{02} = 0 m/s).

After the collision, the incoming ball rebounds straight back at a velocity v_{f1} (3.2 m/s) and the second ball moves at a velocity v_{f2}. Calculate v_{f2}.



Data Table					
m_2	m_1	v_{02}	v_{01}	v_{f1}	v_{f2}
0.45 kg	0.200 kg	0 m/s	8.3 m/s	-3.2 m/s	?

a) The net force acting on the two balls is zero. So, the system is isolated.

c)The direction to the right is the positive direction.

So, V_{01} is positive.

V_{f1} is negative and V_{f2} is positive.

d) The momentum (p) is conserved or remains the same.

Total momentum before the collision = Total momentum after the collision

e) Equation:

Sum of all (m x v) (**before the collision**) = Sum of all (m x v) (**after the collision**)

$$(m_1 \times v_{01}) + (m_2 \times v_{02}) = (m_1 \times v_{f1}) + (m_2 \times v_{f2})$$

(**before the collision**) (**after the collision**)

$$(m_1 \times v_{01}) + (m_2 \times v_{02}) - (m_1 \times v_{f1}) = m_2 \times v_{f2}$$

$$[(m_1 \times v_{01}) + (m_2 \times v_{02}) - (m_1 \times v_{f1})] / m_2 = v_{f2}$$

$$[(0.2 \times 8.3) + (0.45 \times 0) - (0.2 \times (-3.2\ m/s))] / 0.45 = v_{f2}$$

$$[1.66 \quad + 0 \quad + 0.64] \quad / 0.45 = v_{f2}$$

$$2.3 / 0.45 = v_{f2}$$

$$v_{f2} = 5.1\ m/s$$

The ball moves with a speed of 5.1 m/s.

Example 3: Collision of two objects moving in the same direction

(a)A loaded railway car of mass 6000 kg is rolling to the right at 3.0 m/s.
It collides and couples with an empty freight car of mass 3000 kg, rolling to the right on the same track at 2.0 m/s (b). What is the speed and direction of the pair after the collision?

Data Table				
m_2	m_1	v_{02}	v_{01}	v_f
6000 kg	3000 kg	3.0 m/s	2.0 m/s	?

Steps to solve the problem:

a) The net force acting on the two railway cars is zero. So, the system is isolated.

b) This is a sketch of the collision with the two states: (a) for before the collision and (b) for after the collision.

(a) Before
(b) After

c) The direction to the right is the positive direction. So, the direction to the left is negative. The momentum P has the same direction of the velocity.

Then, V_{01} and V_{02} are positive.

d) When the two railways collide, the momentum (p) is conserved or remains the same.

Total momentum before the collision = Total momentum after the collision

e) Equation:

Sum of all (m x v) (**before the collision**) = Sum of all (m x v) (**after the collision**)

$$(m_1 \times v_{01}) + (m_2 \times v_{02}) = (m_1 + m_2) \times v_f$$

$$(3000 \times 2.0) + (6000 \times 3.0) = (9000) \times V_f$$

$$6000 + 18000 = 9000 \times V_f$$

$$24000 = 9000 \times V_f$$

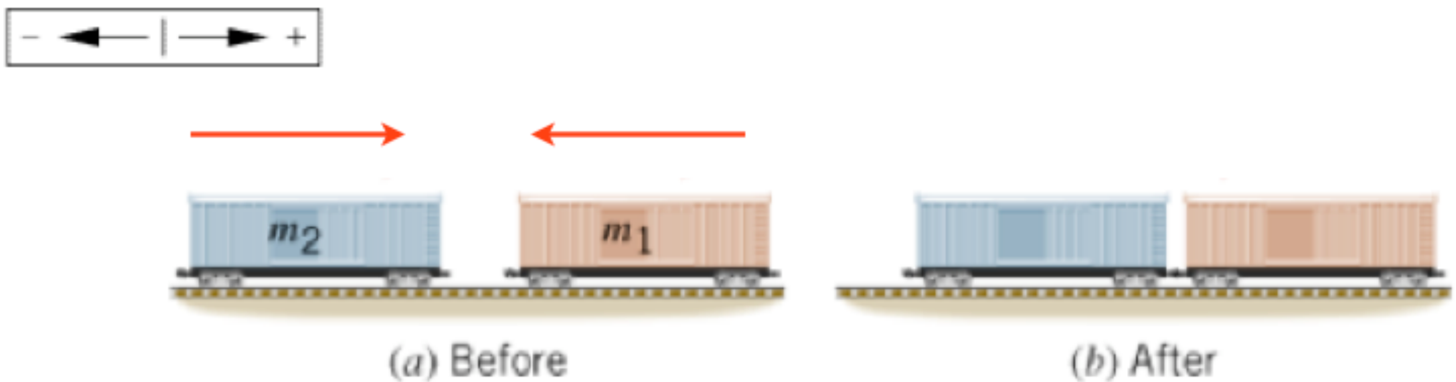
$$V_f = 24000/9000 = 2.66 \text{ m/s}$$

So, the pair couple and moved to the right with a speed of 2.66 m/s.

Example 4: Collision of two objects moving in opposite direction

(a) A loaded railway car of mass 6000 kg is rolling to the right at 2.0 m/s. Another empty freight car of mass 3000 kg is rolling to the left on the same track at 3.0 m/s.

(b) They collide and couple. What is the speed and direction of the pair after the collision?

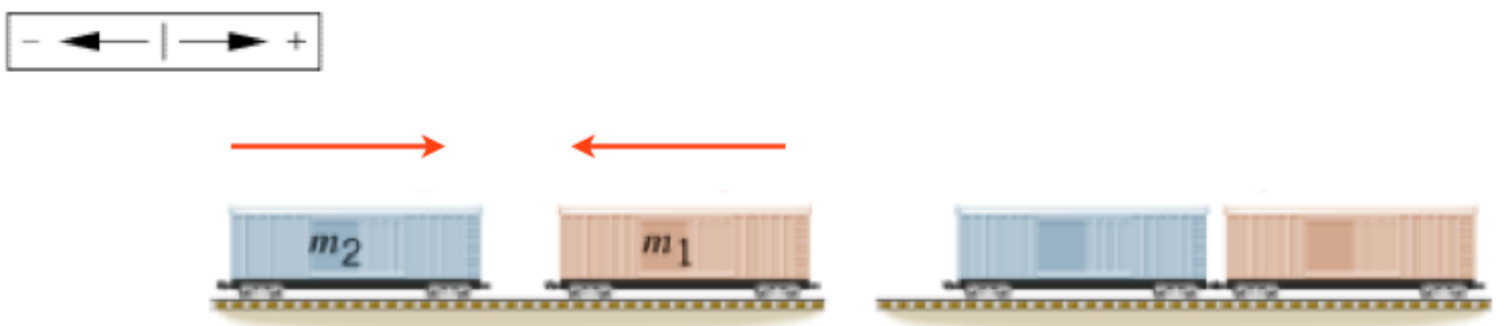


Data Table					
m_2	m_1	v_{02}	v_{01}	v_f	$m_1 + m_2$
6000 kg	3000 kg	2.0 m/s	**- 3.0 m/s**	?	9000 kg

Steps to solve the problem:

a) The net force acting on the two railway cars is zero. So, the system is isolated.

b) This is a sketch of the collision with the two states: (a) for before the collision and (b) for after the collision.



(a) Before (b) After

c) The direction to the right is the positive direction. So, the direction to the left is negative. The momentum P has the same direction of the velocity.

Then, V_{02} is positive and V_{01} is negative.

d) When the two railways collide, the momentum (p) is conserved or remains the same.

Total momentum before the collision = Total momentum after the collision

e) Equation:

Sum of all (m x v) (**before the collision**) = Sum of all (m x v) (**after the collision**)

$$(m_1 \times v_{01}) + (m_2 \times v_{02}) = (m_1 + m_2) \times v_f$$

$$(3000 \times (-3.0)) + (6000 \times 2.0) = (9000) \times V_f$$

$$-9000 + 12000 = 9000 \times V_f$$

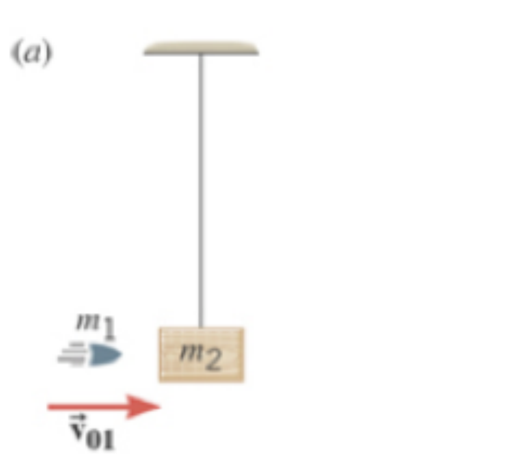
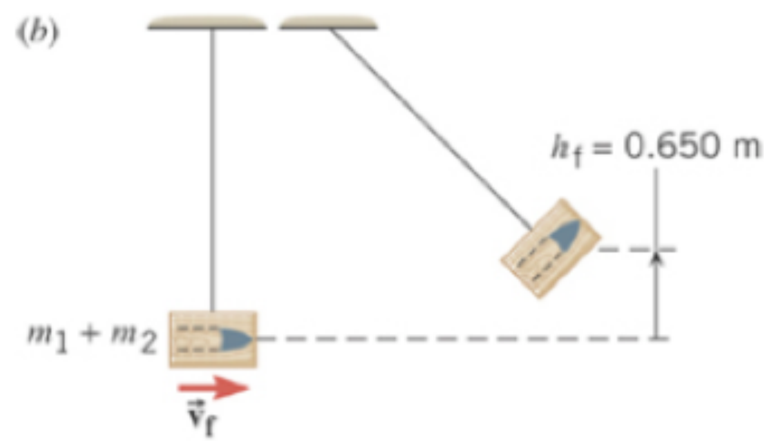
$$3000 = 9000 \times V_f$$

$$V_f = 9000/3000 = 0.33 \text{ m/s}$$

So, the pair couple and moved to the right with a speed of 0.33 m/s

Example 5: Collision of two objects one is moving and the other is at rest.

(a) A bullet of mass (m_1 = 0.0100 kg) is fired with a speed of (v_{01} = 896 m/s) and collide with a block of wood of mass (m_2 = 2.5 kg) at rest (v_{02} = 0 m/s) and suspended by a wire with negligible mass . After the collision (b), they both swing to a height of 0.0650 m. Find the final velocity (v_f) of the wood and the bullet.

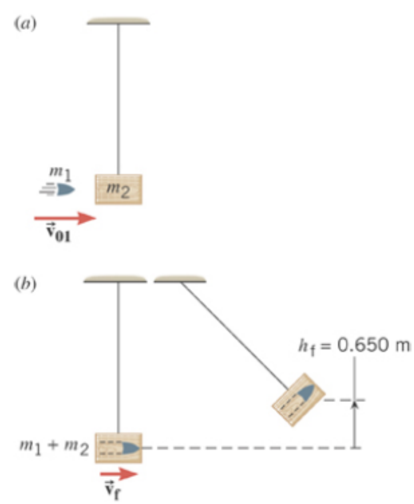
Before	After
	

Data Table					
m_2	m_1	v_{02}	v_{01}	v_f	$m_1 + m_2$
2.5 kg	0.0100 kg	0 m/s	896 m/s	?	2.51 kg

Steps to solve the problem:

a) The net force acting on the bullets and wood is zero. So, the system is isolated.

b)This is the sketch: (a) before the collision and (b) after the collision.



c)The direction to the right is the positive direction.

d)The momentum is conserved:

Total momentum before the collision = Total momentum after the collision

e) Equation:

Sum of all (m x v) (**before the collision**) = Sum of all (m x v) (**after the collision**)

$$(m_1 \times v_{01}) + (m_2 \times v_{02}) = (m_1 + m_2) \times V_f$$

$$(0.010 \times 896) + (2.5 \times 0) = (2.51) \times V_f$$

$$89.6 + 0 = (2.51) \times V_f$$

$$8.96 = (2.51) \times V_f$$

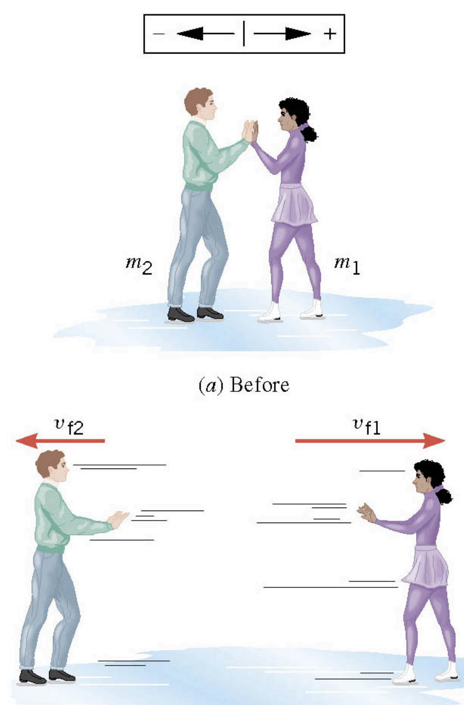
$$V_f = 89.6 / 2.51 = 3.569 \text{ m/s} = 3.57 \text{ m/s}$$

So, after the collision, both the block of wood and the bullet move together to the right with a speed of 3.57 m/s.

Example 6: Starting at rest, two skaters push off against each other.

(a) Before: Starting from rest two skaters push off against each other on ice where friction is negligible. The mass of the woman is m_1 (54 kg) and the mass of the man is m_2 (88 kg).

(b) After: The woman moves away with a velocity $\mathbf{V}_{f1}$ (2.5 m/s) and the man with velocity V_{f2} in the opposite direction. Find the velocity of the man.



(*a*) Before

(*b*) After

Data Table					
m_2	m_1	v_{02}	v_{01}	v_{f1}	v_{f2}
88 kg	54 kg	0 m/s	0 m/s	2.5 m/s	?

a) The net force acting on the two people is zero. So, the system is isolated.

c)The direction to the right is the positive direction.

V_{f2} is negative and V_{f1} is positive.

d) The momentum (p) is conserved.

Total momentum before moving away = Total momentum after moving away

e) Equation:

Sum of all (m x v) (**before**) = Sum of all (m x v) (**after**)

$$(m_1 \times v_{01}) + (m_2 \times v_{02}) = (m_1 \times v_{f1}) + (m_2 \times v_{f2})$$

$$(m_1 \times 0) + (m_2 \times 0) = (54 \times 2.5) + (88 \times v_{f2})$$

$$-(54 \times 2.5) = 88 \times v_{f2}$$

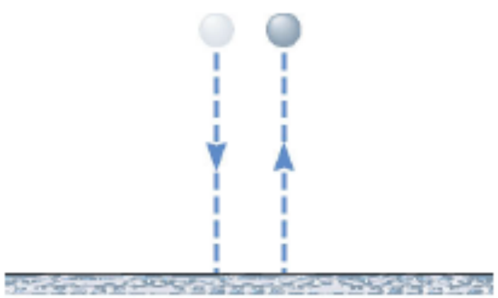
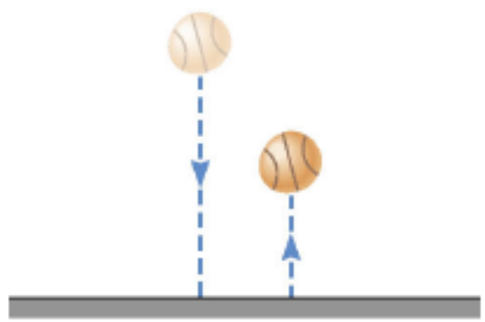
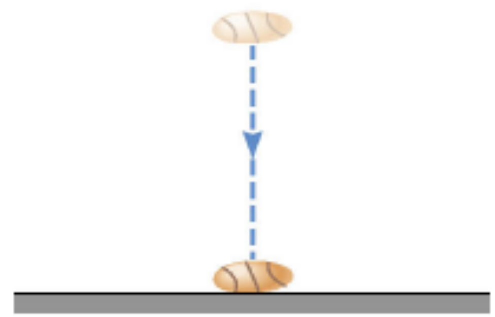
$$v_{f2} = -135 / 88 = -1.5 \text{ m/s}$$

The man moves backward with a speed of 1.5 m/s.

Example 7: The law of conservation of momentum is used to explain how fast does the wreckage move immediately after the collision. Conservation of momentum works just as well to describe explosions.

Example 8:

In any type of collision, the momentum is always conserved for an isolated system.

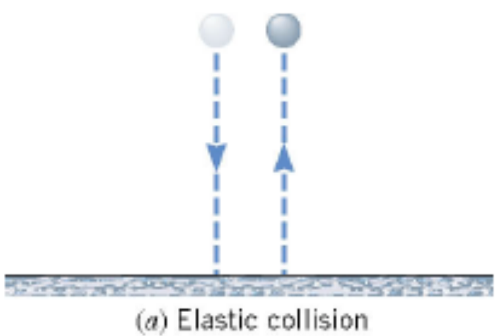
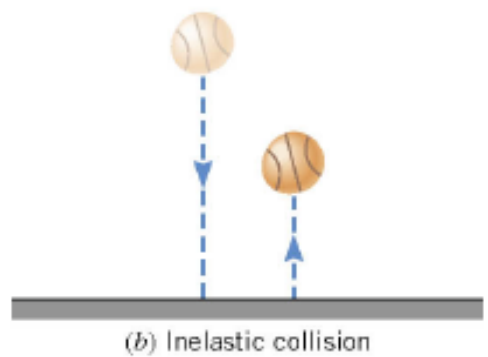
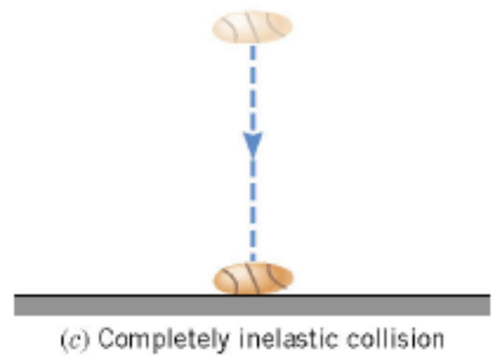
Elastic Collision	Inelastic Collision	Completely Inelastic Collision
 (a) Elastic collision	 (b) Inelastic collision	 (c) Completely inelastic collision
$1/2\ (m \times v_i^2) = 1/2\ (m \times v_f^2)$	$1/2\ (m \times v_i^2)$ # $1/2\ (m \times v_f^2)$	$v_f = 0$ m/s

v_f ; speed after collision v_i ; speed before collision   m; mass of ball

Types of Collisions and Kinetic Energy

Elastic collision -- One in which the total kinetic energy [1/2 (m x v^2)] of the system after the collision is equal to the total kinetic energy before the collision.

Inelastic collision -- One in which the total kinetic energy of the system after the collision is not equal to the total kinetic energy before the collision; if the objects stick together after colliding, the collision is said to be completely inelastic.

Elastic Collision	Inelastic Collision	Completely Inelastic Collision
		
1/2 (m x v_i^2) = 1/2 (m x v_f^2)	1/2 (m x v_i^2) # 1/2 (m x v_f^2)	v_f = 0 m/s

v_f ; speed after the collision v_i ; speed before the collision m; mass of the ball

Practice Problems

a) A 5000 kg boxcar moving at 5.2 m/s on a level, frictionless track runs into a stationary 8000 kg tank car. If they hook together in the collision, how fast will they be moving afterwards?

(Answer: 2.0 m/s [forwards])

b) A 0.1 kg ball moving at a constant velocity of 0.2 m/s strikes a 0.4 kg ball that is at rest. After the collision, , the first ball rebounds straight back at 0.12 m/s. Calculate the final velocity of the second ball.

(Answer: 0.08 m/s [forwards])

References:

1) Humanic. (2013). www.physics.ohio-state.edu/~humanic/. In Thomas Humanic Brochure Page.

Physics 1200 Lecture Slides: Dr. Thomas Humanic, Professor of Physics, Ohio State University, *2013-2014 and Current.* *www.physics.ohio-state.edu/~humanic/*

2) Cutnell, J. D. & Johnson, K. W. (1998). *Cutnell & Johnson Physics, Fourth Edition*. New York: John Wiley & Sons, Inc.

The edition was dedicated to the memory of Stella Kupferberg, Director of the Photo Department: "We miss you, Stella, and shall always remember that a well-chosen photograph should speak for itself, without the need for a lengthy explanation"

3) Martindale, D. G. & Heath, R. W. & Konrad, W. W. & Macnaughton, R. R. & Carle, M. A. (1992). *Heath Physics.* Lexington: D.C. Heath and Company

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5) Schnick, W.J. (n.d.). *Calculus-based physics, A Free Physics Textbook*. Retrieved from http://www.anselm.edu/internet/physics/cbphysics/index.html

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