

Impulse and Momentum

by

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P3.4g Explain how the time of impact can affect the net force (e.g., air bags in cars, catching a ball).

Items:

- 1- Impulse
- 2- Linear Momentum and Impulse
- 3- Impulse-Momentum Theorem
- 4- Factors Affecting Impulse

IMPULSE (J)

Impulse (impulsive force) is a force acting for a very short period of time, as in impact or collision between objects.

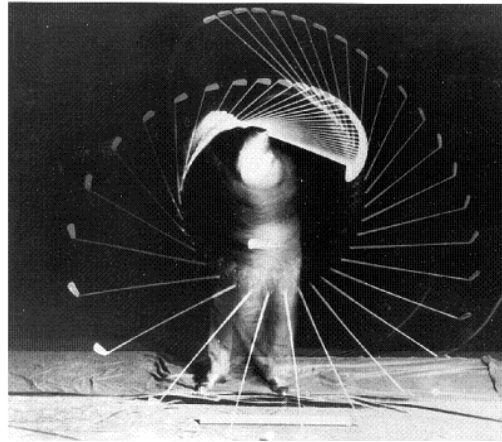
Impulse is the product of the net average force and the time interval for which it is applied. It has the symbol (J) and the unit Newton. Second (N.S).

Impulse is a vector quantity and has the same direction the net average force (F) causing the impulse.

Impulse
$\vec{J} = \text{impulse} = \vec{F}\Delta t$
unit: N.S

Example 1: Hitting a Golf Ball.

What is the impulse given to a golf ball by a club if they are in contact for 0.005 second, during which time the club exerted an average force of 500 N on the ball?



Data Table		
F	J	Δt
500 N	?	0.005 s

$$J = \text{impulse} = F\Delta t = 500 \times 0.005 = 2.5 \text{ N}\cdot\text{s} \text{ [forward]}$$

Linear Momentum (P)

The momentum of an object is a measure of how hard it is to stop that object.

The linear momentum (P) of an object is equal to the product of the object's mass (m) and velocity (v).

It is a vector quantity and has the same direction as the velocity.

Linear Momentum (p)

$$\vec{p} = m\vec{v}$$

Unit: (kg . m/s)

Impulse - Momentum Theorem

When a net force F acts on an object, the impulse of the net force is equal to the change in momentum that it produces on the object.

Impulse = Change of momentum

Impulse (J) = Final momentum (P_f) - initial momentum (P_o)

Impulse-Momentum Theorem

Impulse (J) = Final momentum (P_f) - initial momentum (P_o)

$$\vec{J} = \vec{P}_f - \vec{P}_o$$

$$\vec{F} \Delta t = m\vec{V}_f - m\vec{V}_o$$

$$\vec{F} = m\vec{V}_f - m\vec{V}_o / \Delta t$$

F is the average net force acting on the object, in Newton.

Δt is the time interval for which the force acts, in second.

m is the mass of the object, in kilogram (kg)

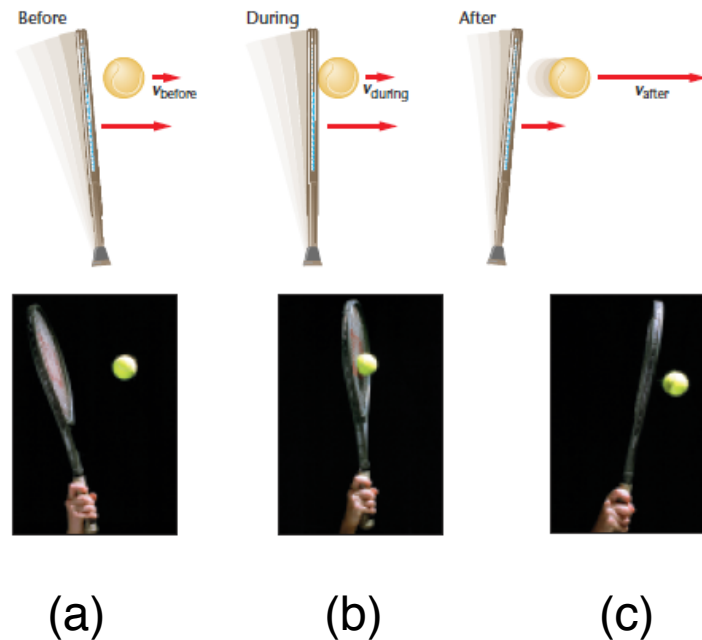
V_f is the final velocity of the object after the impact or collision, in (m/s)

V_o is the initial velocity of the object before the impact or collision, in (m/s)

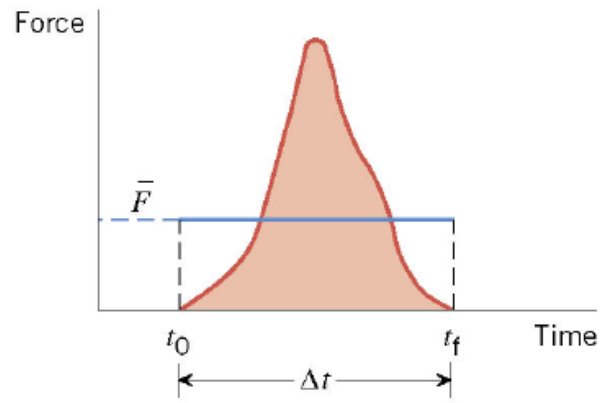
A large change in momentum occurs when there is a large change of impulse. Therefore, to change the momentum of an object, the impulse needs to be changed.

Analysis of the Impulse - Momentum Theorem

Below is a picture of a racquet before, during and after the collision with a tennis ball. How are the velocities of the ball before and after the collision related to the force (F) acting on it?



During this process, the force (F) of the racquet increases to a maximum then decreases to zero during an interval of time Δt as shown in the figure below:



- (a) Before the hit: The velocity of the ball is approaching with V before. The racquet force increases from zero to hit the ball. The ball has a momentum P_1 .
- (b) During the hit or collision. The racquet hits the ball with maximum force.
- (c) After the hit: The ball leaves with more velocity. The racquet force decreases to zero. The momentum of the ball is now P_2 that is different than P_1 .

F is the magnitude of the average force: Impulse = $\mathbf{J} = F\Delta t$

So, the racquet applies an impulse on the tennis ball. As a result, the tennis ball leaves with greater velocity and its momentum changes.

According to Newton's Second Law of Motion

$$\vec{F} = m \vec{a} \quad (1)$$

Acceleration = change in velocity/time interval

$$a = \frac{\Delta \vec{v}}{\Delta t} = \frac{(\vec{v}_f - \vec{v}_o)}{\Delta t} \quad (2)$$

($\vec{v}_f = \vec{v}_{\text{after}}$, $\vec{v}_o = \vec{v}_{\text{before}}$)

Insert the value of acceleration in equation (1)

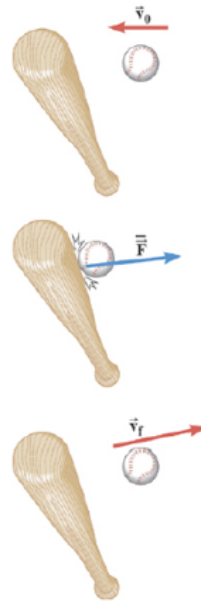
$$\vec{F} = m \frac{(\vec{v}_f - \vec{v}_o)}{\Delta t}$$

$$\vec{F} \Delta t = m (\vec{v}_f - \vec{v}_o)$$

$$\vec{F} \Delta t = m \vec{v}_f - m \vec{v}_o = \Delta \vec{p}$$

This is the Impulse - Momentum Theorem

Example 2: Hitting a Pitched Baseball: When a bat hits a ball, an average force F is applied to the ball. As a result, the ball's velocity changes from an initial value V_o to a final value of V_f



First picture (top): The mass of the baseball is 0.14 kg, and the initial velocity (V_o) is -38 m/s (negative: west direction).

Second picture (middle): The bat applies an average force (F) and hits the ball.

Third picture (bottom): The ball departs with a final velocity (V_f) of +58 m/s.

The time of contact with the bat (Δt) is 1.6×10^{-3} s. What is value of the average force (F)?

Data Table				
F	v_f	v_o	m	Δt
?	58 m/s	-38 m/s	0.14 kg	1.6×10^{-3} s

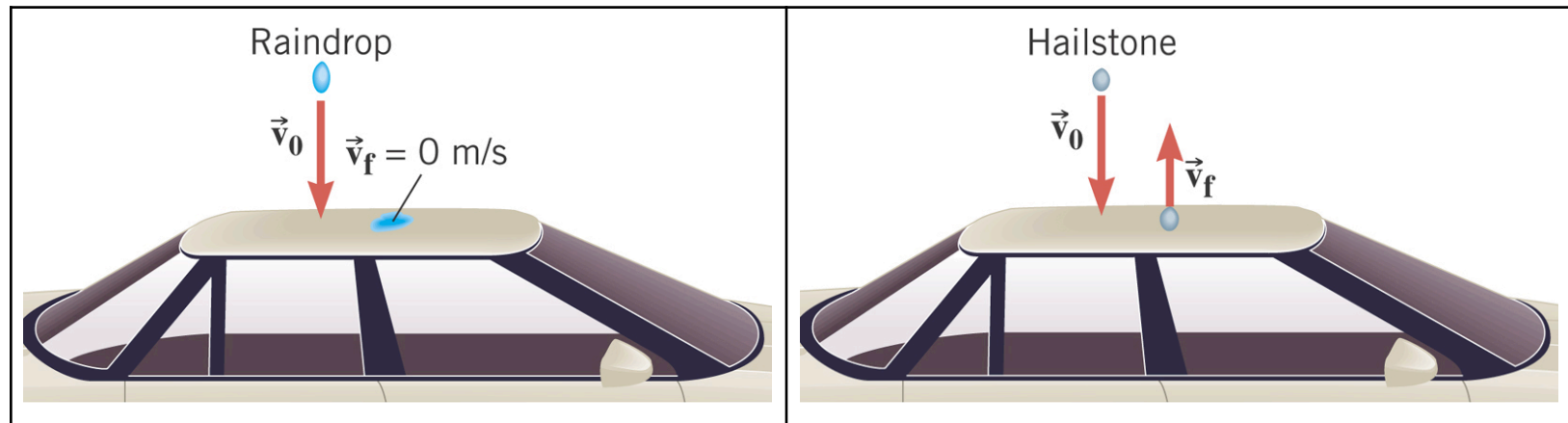
$$\vec{F} \Delta t = m\vec{v}_f - m\vec{v}_o = (0.14)(58) - (0.14)(-38) = + 13.4 \text{ kg m/s}$$

$$\vec{F} = m\vec{v}_f - m\vec{v}_o / \Delta t = (0.14)(58) - (0.14)(-38) / (1.6 \times 10^{-3}) = +8400 \text{ N}$$

So, the bat hits the ball with a force of 8400 Newtons and changes the direction and momentum of the ball.

Example 3: Raindrops versus Hailstones Falling on a Car Roof.

During a rain storm, the raindrops and the hailstones fall in exactly the same way. They both fall with the same initial velocity (V_o) and mass rate in the same time interval. They both strike the roof perpendicularly. The hailstones often damages cars more than a rainstorm does.



The raindrop comes to a stop (final velocity $V_f = 0 \text{ m/s}$), while the hailstone changes its velocity from V_o downward to V_f upward. Therefore, the change in momentum for the hailstone is greater. Consequently, a greater impulse acts on the hailstone. The car roof experiences a greater force. (*Action-Reaction*)

Important factors that affect the impulse and consequently the momentum.

$$J = F \Delta t$$

To change the impulse, two quantities are important - the force applied to the object and the length of time the force is applied.

A large impulse is caused by either a large force acting over a short period of time or a small force acting over a long period of time.

Angular impulse - Momentum Theorem

Angular impulse is the change of angular momentum

Example 4: Airbags: Using the impulse-momentum theorem to save lives.



In a collision, The car and its passengers change momentum rapidly.
A large impulse brings the car to a stop.

$$\vec{F}\Delta t = \text{impulse} = \vec{J}$$

For a certain value of impulse, in a car where a passenger is not protected by seat belt or an air bag, the passenger might strike the hard dashboard and come to stop very quickly-possibly within 0.7 seconds.

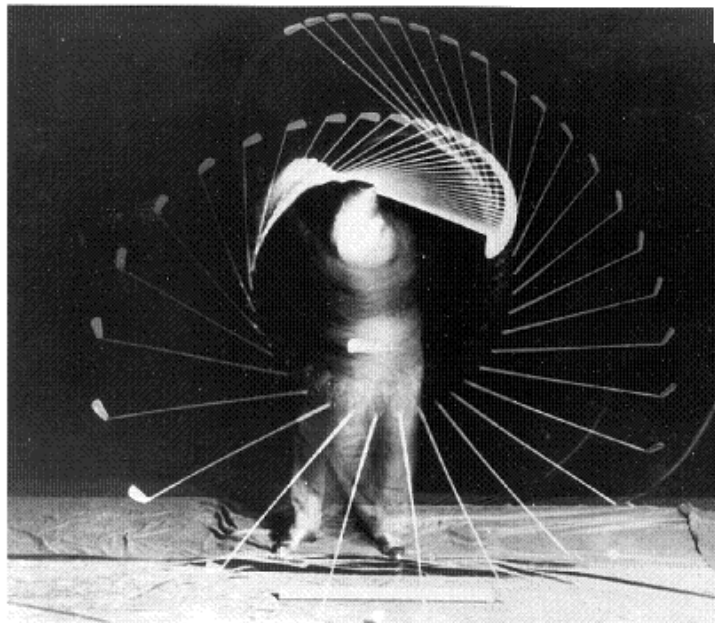
In a similar collision a passenger protected by an air bag would undergo the same change in momentum and impulse. But the inflated air bag brings the passenger to a stop much more slowly (Δt).

Why?

The impulse or product $F\Delta t$ is the same in both cases, with the airbag or without the airbag. In the case of airbag, the greater Δt means that F will be much smaller. So the force (F) exerted on the driver will be greatly reduced by the air bag.

Example 5: Hitting a Golf Ball.

To give a golf ball its maximum momentum, you must increase the impulse. Therefore, you must hit the ball hard and follow through with your swing. Following through on the swing keeps the club in contact with the ball as long as possible. This increases the time the force is exerted on the ball, which increases the impulse and consequently increases the momentum. So, the ball leaves with its maximum velocity.



Practice Exercises:

1) What impulse is exerted in each of the following cases?

a) a force of 25 N[E] on a dynamic cart for 3.2 s.

(Answer: 80 N.s [E])

b) a hockey stick exerting a force of 120 N on a puck during the 0.05 s they are in contact.

(Answer: 6 N.s [forwards])

c) the Earth pulling down on a 12 kg rock during 3.0 s it takes to fall from a cliff.

(Answer: 3.5×10^2 N.s [down])

2) A billiard ball of mass 200 g rolls towards the right hand cushion of the billiard table at 2.0 m/s and rebounds straight back at 2.0 m/s/

a) What is the change in momentum as a result of hitting the cushion?

(Answer: - 0.80 kg. m/s [right])

b) What impulse is give to the ball by the cushion?

(Answer: 0.80 kg. m/s [left])

3) A 2.0 kg skateboard is rolling across a smooth flat floor, when a small girl kicks it, causing it to speed up to 4.5 m/s in 0.50 s without changing direction. If the average force exerted by the girl on the skateboard in its direction of motion was 6.0 N, with what initial velocity was it moving?

(Answer: 3.0 m/s [forwards])

4) What average force will stop a 100 kg car in 1.5 s if the care is moving at 22 m/s?

(Answer: -1.5 x 10⁴ N [backwards])

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The edition was dedicated to the memory of Stella Kupferberg, Director of the Photo Department: “We miss you, Stella, and shall always remember that a well-chosen photograph should speak for itself, without the need for a lengthy explanation”

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