Parallel Circuits

by

Nada Saab-Ismail, PhD, MAT, MEd, IB

e-mail:saab1055@gmail.com

P4.10g Compare the currents, voltages, and power in parallel and series circuits.

P4.10B Identify common household devices that transform electrical energy to other forms of energy, and describe the type of energy transformation.

P4.10C Given diagrams of many different possible connections of electric circuit elements, identify complete circuits, open circuits, and short circuits and explain the reasons for the classification.

Items;

1- Parallel Circuits

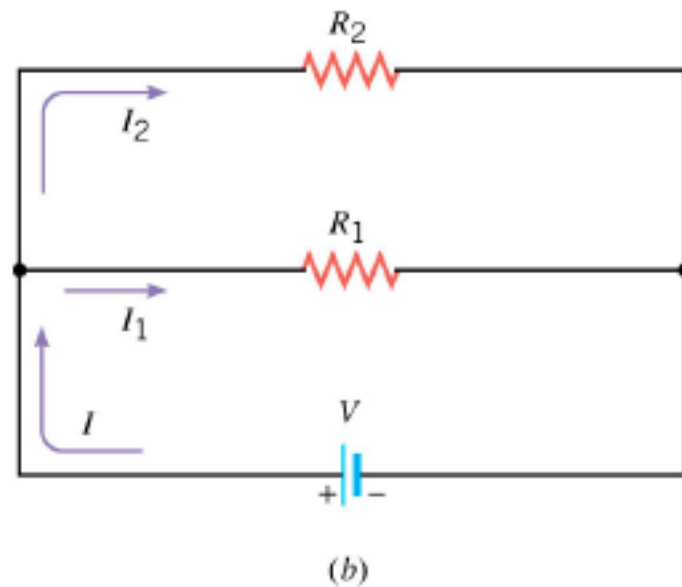2- Equivalent Voltage

3- Equivalent Resistor

Parallel Circuits

In a parallel circuit, the electrons have a choice of several paths through the circuit, as a result, may pass through any one of the several loads in the circuit. Parallel wiring means that the devices are connected in such a way that the same voltage is applied across each device.
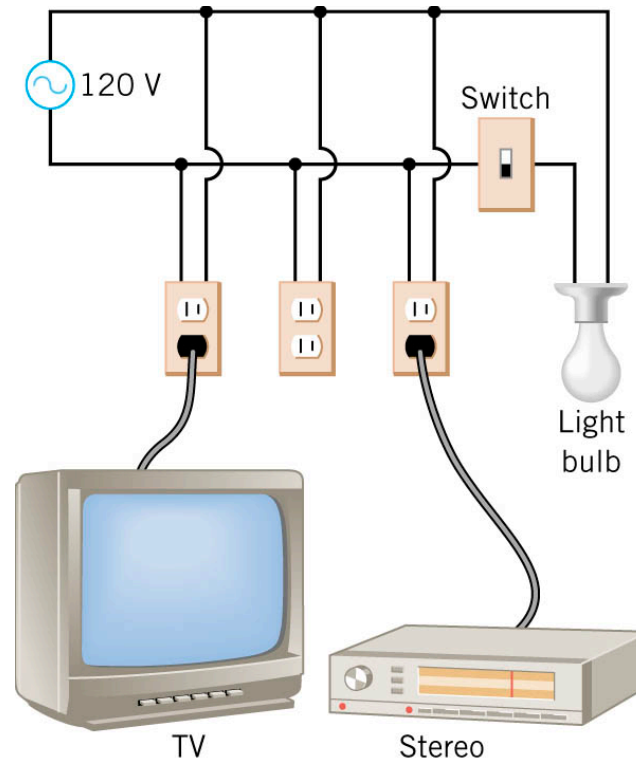

(a)
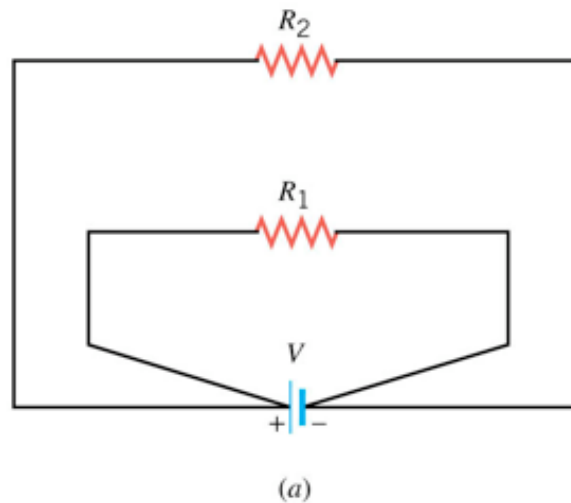
Since R_1 and R_2 are connected in parallel, they have the same voltage (V) across them.

When two resistors are connected in parallel, each receives current from the battery as if the other were not present. Therefore, the two resistors connected in parallel draw more current than does either resistor alone.
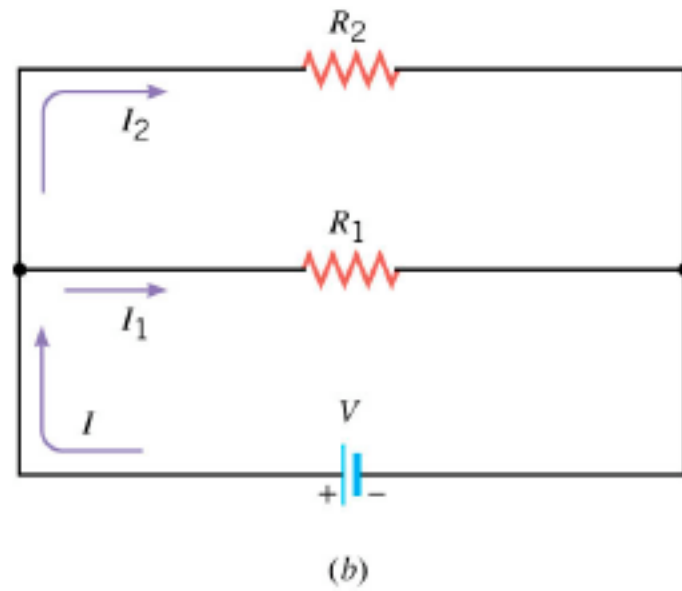

(b)

Example 1: *Household Outlets;*

Household outlets are connected in parallel instead of series so that each appliance sees the same voltage; also, if one appliance blows out, power is not cut to all appliances.

Equivalent Current and Resistance in Parallel Circuit

(b)

Ohm's Law:

$$V = I\,R$$

$$I = V/R$$

According to Kirchhoff's junction rule:

sum of into currents = sum of currents out

$$I = I_1 + I_2$$

$$V/R_p = V/R_1 + V/R_2 = V(1/R_1 + 1/R_2)$$

So,

$$R_p = 1/R_1 + 1/R_2$$

R_p is the equivalent resistance of the parallel circuit (parallel resistors).

Equivalent Resistance in Parallel Circuit
$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

Two parallel resistors R_1 and R_2 have a single equivalent resistor (R_p) that is smaller than either R_1 or R_2.

Analogy, Additional Insight to Parallel Wiring

The two parallel pipe sections in figure (a) are equivalent to a single pipe of the same length and same total cross sectional area in figure (b).

Cross-sectional
area = A_2

Cross-sectional
area = $A_1 + A_2$
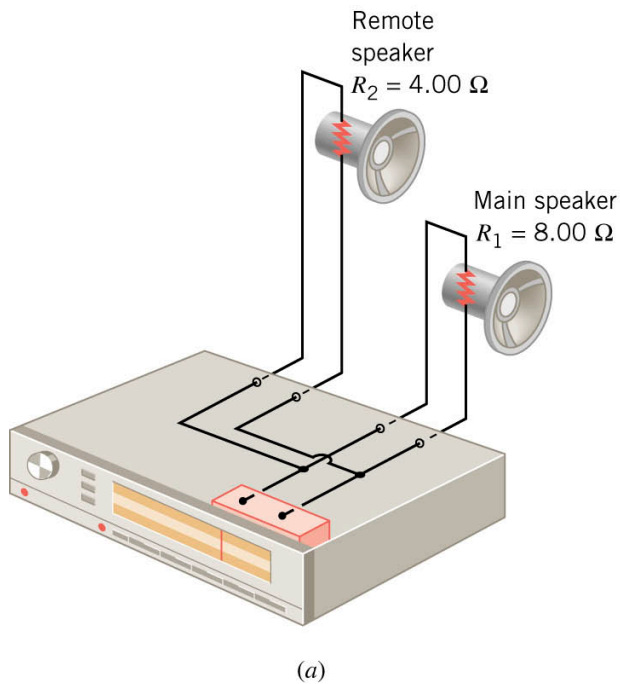
Cross-sectional
area = A_1

Pump

Pump

(a)

(b)
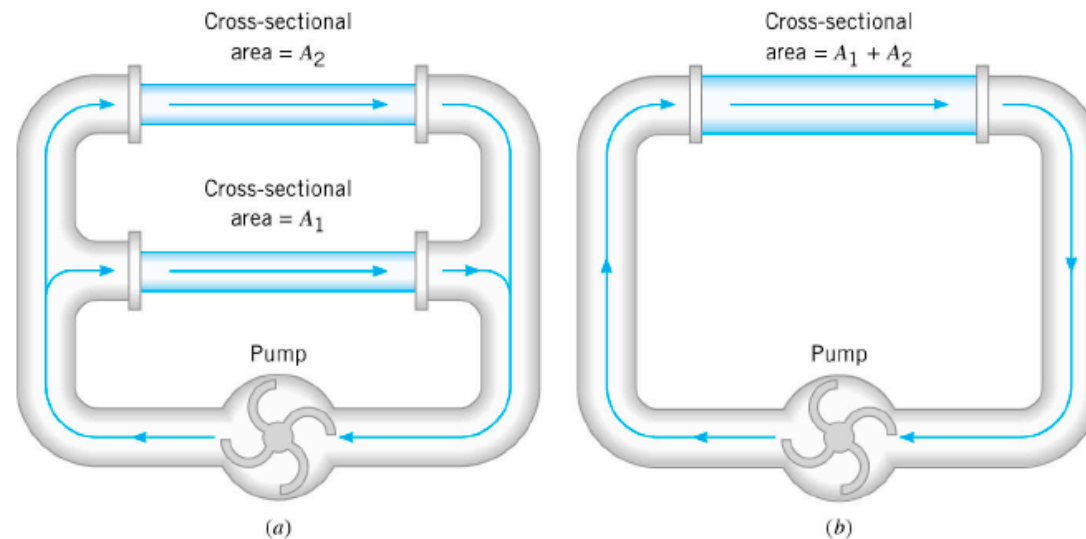
The pipe in part (b) offers less resistance to the water flow than either of the narrower pipes offers individually in (a). Therefore, the pipe in (b) can push through more water per second. That can explain both concepts: "A larger current arises from a smaller resistance" and "Two parallel resistors R_1 and R_2 have a single equivalent resistor (R_p) that is smaller than either R_1 or R_2."
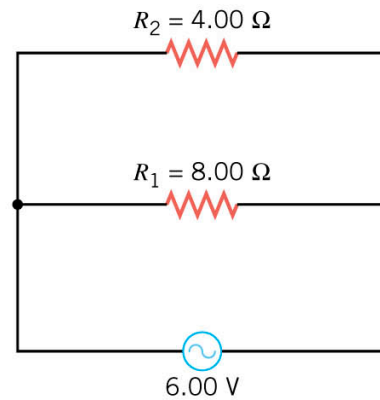
Example 2: *Equivalent Resistance in Parallel Circuit;*

The figure shows a circuit C1 composed of a 24-V battery and four resistors: 110, 180 and 470 Ω . Its equivalent circuit C2 has two resistors: 110 and 130 Ω .

(b)

C1   C2

The two parallel resistors 180 and 470 Ω in C1 can be replaced by an equivalent resistor of 130 Ω in C2.

$$1/R_p = 1/R_1 + 1/R_2 = 1/180 + 1/470$$
$$R_p = 130 \ \Omega$$

Example 3: *Main and Remote Stereo Speakers;*

Most receivers allow the user to connect to "remote" speakers in addition to the main speakers. At the instant represented in the picture, the voltage across the speakers is 6.00 V in the circuit schematic representation C1.
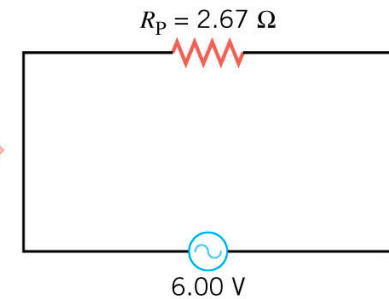
Remote speaker
$R_2 = 4.00 \ \Omega$

Main speaker
$R_1 = 8.00 \ \Omega$

$R_2 = 4.00 \ \Omega$

$R_1 = 8.00 \ \Omega$

6.00 V

Is equivalent to

$R_P = 2.67 \ \Omega$

6.00 V

(a)

(b)

C1

C1

C2

Determine:

   (a) the equivalent resistance of the two speakers in the circuit representation C2,

   (b) the total current supplied by the receiver,

   (c) the current in each speaker, and

   (d) the power dissipated in each speaker.

Data Table							
R_1	R_2	V	R_p	I_{total}	$I_{for\ R1}$	$I_{for\ R2}$	P_{total}
4.00 Ω	8.00 Ω	6.00 V	?	?	?	?	?

(a) The two parallel resistors 4.00 and 8.00 Ω in C1 can be replaced by an equivalent resistor of 2.67 Ω.

$$\frac{1}{R_p} = \frac{1}{8.00\ \Omega} + \frac{1}{4.00\ \Omega} = \frac{3}{8.00\ \Omega} \qquad \rightarrow \qquad R_p = 2.67\ \Omega$$

(b)

$$I_{rms} = \frac{V_{rms}}{R_p} = \frac{6.00\ V}{2.67\ \Omega} = 2.25\ A$$

(c)

$$R_1: \quad I_{rms} = \frac{V_{rms}}{R} = \frac{6.00\ V}{8.00\ \Omega} = 0.750\ A \qquad\qquad R_2: \quad I_{rms} = \frac{V_{rms}}{R} = \frac{6.00\ V}{4.00\ \Omega} = 1.50\ A$$

I_{tot} = 0.750 A + 1.50 A = 2.25 A, as in part (b)

(d)

$$R_1: \quad \overline{P} = I_{rms}V_{rms} = (0.750\ A)(6.00\ V) = 4.50\ W$$

➔ P_{tot} = 4.50 W + 9.00 W = 13.5 W

$$R_2: \quad \overline{P} = I_{rms}V_{rms} = (1.50\ A)(6.00\ V) = 9.00\ W$$

Alternatively,
$P_{tot} = I_{tot}V$ = (2.25)(6.00) = 13.5 W

Example 4: *Three-Way Light Bulb and Parallel Wiring;*

Within the bulb there are two separate filaments. When one burns out, the bulb can produce only one level of illumination, but not the highest. The filaments connected in parallel.

100-W filament

50-W filament

Simplified version
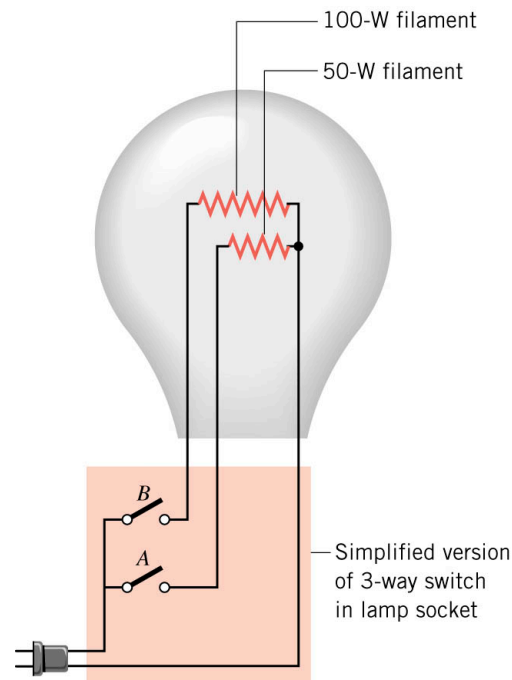of 3-way switch
in lamp socket

B

A
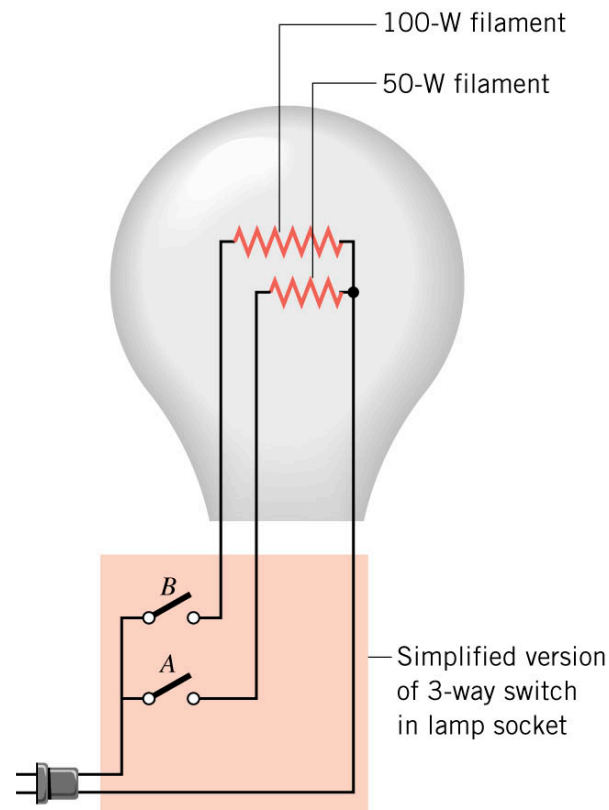
The two filaments are used to produce three different illumination levels (1, 2 and 3) with output power of 50, 100 and 150 W.

Level 1: A on, B off --> 50 W output

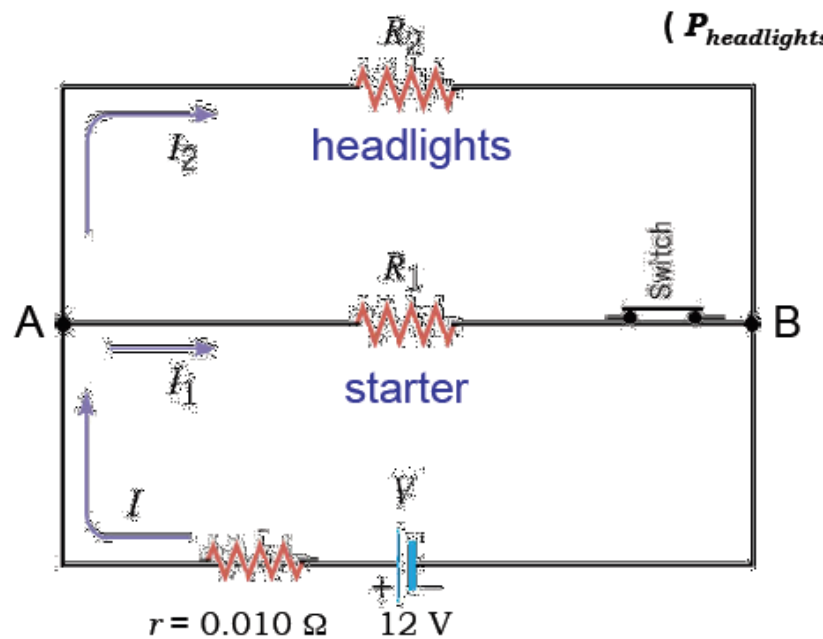Level 2: A off, B on --> 100 W output

Level 3: A on, B on --> 50 W + 100 W = 150 W output

100-W filament

50-W filament

B

A

Simplified version
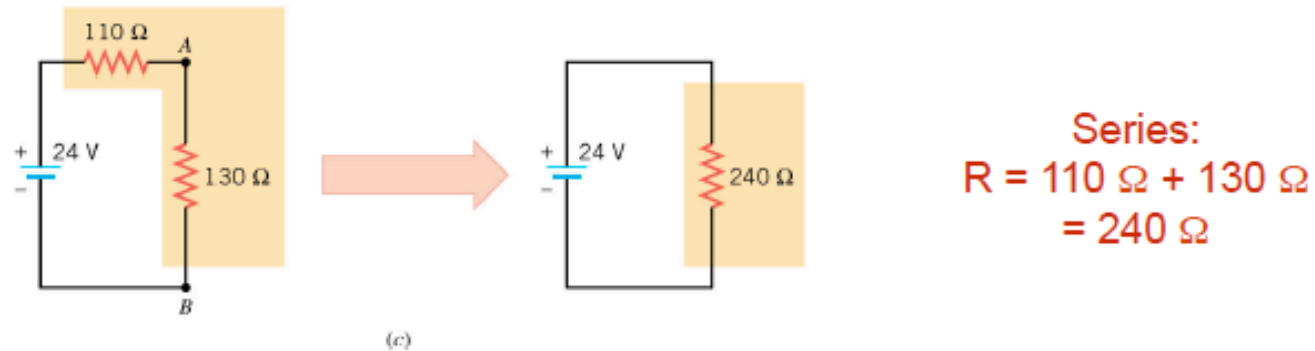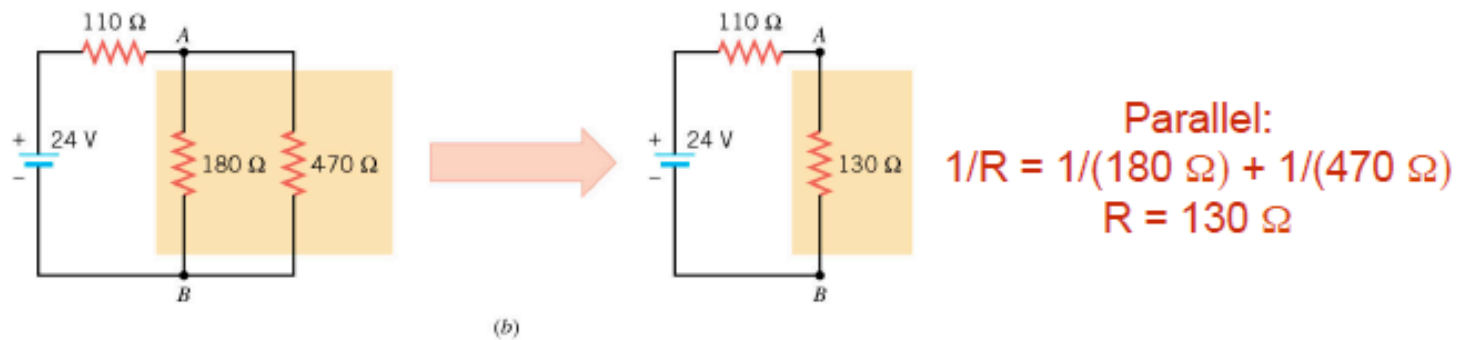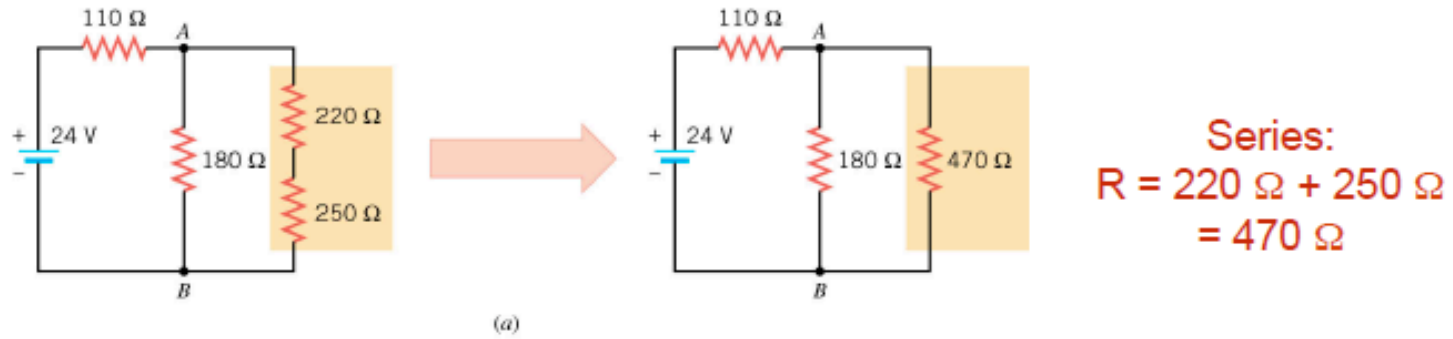of 3-way switch
in lamp socket

Example 5: *Why do your headlights dim when you start up your car?*

Before turning on the starter switch, $I_1 = 0$ A.

After turning on the switch, a large current flow through I_1. The headlights has a reduced potential, so they dim.

Example 6: *Circuits Wired Partially in Series and Partially in Parallel.*

(a)

Series:
R = 220 Ω + 250 Ω
= 470 Ω

(b)

Parallel:
1/R = 1/(180 Ω) + 1/(470 Ω)
R = 130 Ω

(c)

Series:
R = 110 Ω + 130 Ω
= 240 Ω

References:

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Physics 1200 Lecture Slides: Dr. Thomas Humanic, Professor of Physics, Ohio State University, *2013-2014 and Current. www.physics.ohio-state.edu/~humanic/*

2) Cutnell, J. D. & Johnson, K. W. (1998). *Cutnell & Johnson Physics, Fourth Edition*. New York: John Wiley & Sons, Inc.

The edition was dedicated to the memory of Stella Kupferberg, Director of the Photo Department: "We miss you, Stella, and shall always remember that a well-chosen photograph should speak for itself, without the need for a lengthy explanation"

3) Martindale, D. G. & Heath, R. W. & Konrad, W. W. & Macnaughton, R. R. & Carle, M. A. (1992). *Heath Physics*. Lexington: D.C. Heath and Company

4) Zitzewitz, P. W. (1999). *Glencoe Physics Principles and Problems*. New York: McGraw-Hill Companies, Inc.

5) Schnick, W.J. (n.d.). *Calculus-based physics, A Free Physics Textbook*. Retrieved from http://www.anselm.edu/internet/physics/cbphysics/index.html

6) Nada H. Saab (Saab-Ismail), (2010-2013) Westwood Cyber High School, Physics, Detroit, Michigan, U.S.A.

7) Nada H. Saab (Saab-Ismail), (2009- 2014) Wayne RESA, Bilingual Department, Detroit, Michigan, U.S.A.