## Pendulum

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P4.3A Identify the form of energy in given situations (e.g., moving objects, stretched springs, rocks on cliffs, energy in food).
P4.3B Describe the transformation between potential and kinetic energy in simple mechanical systems (e.g., pendulums, roller coasters, ski lifts).

## Items;

1- Pendulum
2- Period of Pendulum
3- Transformation of Energy
4- Period of Planets

## Pendulum

T is the period of the pendulum. It is the time for one cycle measured in seconds. One complete cycle is one period.
$L$ is the length of the pendulum, measured from top end of the string to the center of the bob, in meters.


The period of a pendulum is given by the following equation:

$g$ is the acceleration due to gravity. The unit of $g$ is meter per second square.
On the Earth, $g$ up is $9.8 \mathrm{~m} / \mathrm{s}^{2}, g$ down is $-9.8 \mathrm{~m} / \mathrm{s}^{2}$
$\cdot \Pi=3.14$. It is a constant

A pendulum 1.0 m long is used in some grandfather clocks. Each half cycle marks on second. A clockwork motor gives the pendulum a small push at the right moment during each swing, to keep it going.


## Example 1: Grandfather Clock.

A pendulum 1.0 m long is used in a grandfather clock. What is the period of a pendulum 1.0 m long?

$$
\begin{aligned}
T & =2 \pi \sqrt{\frac{L}{g}} \\
& =2 \pi \sqrt{\frac{1.0 \mathrm{~m}}{9.8 \mathrm{~m} / \mathrm{s}^{2}}} \\
& =2 \pi \sqrt{0.102 \mathrm{~s}^{2}} \\
& =2(3.14)(0.319 \mathrm{~s}) \\
& =2.0 \mathrm{~s}
\end{aligned}
$$

The period of a pendulum 1.0 m long is 2.0 seconds.

## Pendulum, <br> Transformation of Energy Conservation of Mechanical Energy

The transformation between potential and kinetic energy in the simple oscillation of a pendulum bob is shown in figures a and b (below).


Energy Versus Position


Figure b, is the energy versus position graph. It shows the transformation between the potential energy ( Ug , red graph) and kinetic energy ( K , blue graph) as well as the conservation of the total mechanical energy ( $K+U g$, purple line) in positions A, B and C. Notice that the value of the mechanical energy remains constant as depicted by the straight purple line.

Energy Versus Position


## Period of a Planet

A planet of mass $m$ is revolving around the sun of mass $M$, the centripetal force is

$$
\mathrm{F}_{c}=\mathrm{m} \mathrm{a}_{c}
$$

$a_{c}$ : centripetal acceleration;

Using Newton's Law of Universal Gravitation, the force of attraction:

$$
\mathrm{F}_{c}=G m M / r^{2}
$$

$r=$ distance to the sun

$$
\begin{aligned}
m a_{c} & =G m M / r^{2} \\
a_{c} & =G M / r^{2}
\end{aligned}
$$

we know that :

$$
T \approx 2 \pi \sqrt{\frac{L}{g}}
$$

In this case:

$$
\begin{gathered}
g=a_{c}=G M / r^{2} \\
L=r
\end{gathered}
$$

Replacing $g$ and $L$ by their values in the formula of period:

$$
T \approx 2 \pi \sqrt{\frac{L}{g}}
$$

| Orbital Period |
| :---: |
| $\tau=2 \pi \sqrt{\frac{r^{3}}{G M}}$ |

$\mathrm{G}=6.673 \times 10^{-11} \mathrm{~N} . \mathrm{m}^{2} / \mathrm{kg}^{2}$
$\mathrm{M}=$ Mass of the sun $1.99 \times 10^{30} \mathrm{~kg}$

## Example 2: Orbital Period of Earth

The distance from Earth to Sun is $1.5 \times 10^{11} \mathrm{~m}$. Find the orbital period of Earth.


| Data Table |  |  |  |
| :---: | :---: | :---: | :---: |
| $r$ | $G$ | $M \operatorname{sun}$ | $t$ |
| $1.5 \times 10^{11} \mathrm{~m}$ | $6.673 \times 10^{-11}$ | $1.99 \times 10^{30} \mathrm{~kg}$ | $?$ |

The orbital period is:

$$
\begin{gathered}
\tau=2 \pi \sqrt{\frac{r^{3}}{G M}} \\
\tau_{\text {Earth }}=2 \pi \sqrt{\frac{\left(1.5 \times 10^{11}\right)^{3}}{\left(6.67 \times 10^{-11}\right)\left(1.99 \times 10^{30}\right)}}=3.2 \times 10^{7} \mathrm{~s}
\end{gathered}
$$

One year is 365 days $=(365 \times 24 \times 60 \times 60)$ second $=3.1563000$ s or $3.2 \times 10^{7} \mathrm{~s}$

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