Ideal Spring

by

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P4.3A Identify the form of energy in given situations (e.g., moving objects, stretched springs, rocks on cliffs, energy in food).

P4.3B Describe the transformation between potential and kinetic energy in simple mechanical systems (e.g., pendulums, roller coasters, ski lifts).

Items;

1- Hooke’s Law, Spring

2- Elastic Force

3- Elastic Potential Energy

4- Elastic Work

5- Conservation of Mechanical Energy

Hooke’s Law, Spring

In 1978, the British scientist Robert Hooke was one of the first to study the elasticity of matter and published his law: “The amount of deformation of an elastic object is proportional to the force applied to deform it”. “Stress is proportional to strain”.

Hooke’s Law
$F = Kx$

F is the force exerted on the deformed spring, in newtons

x is the amount of deformation of the spring, in meters.

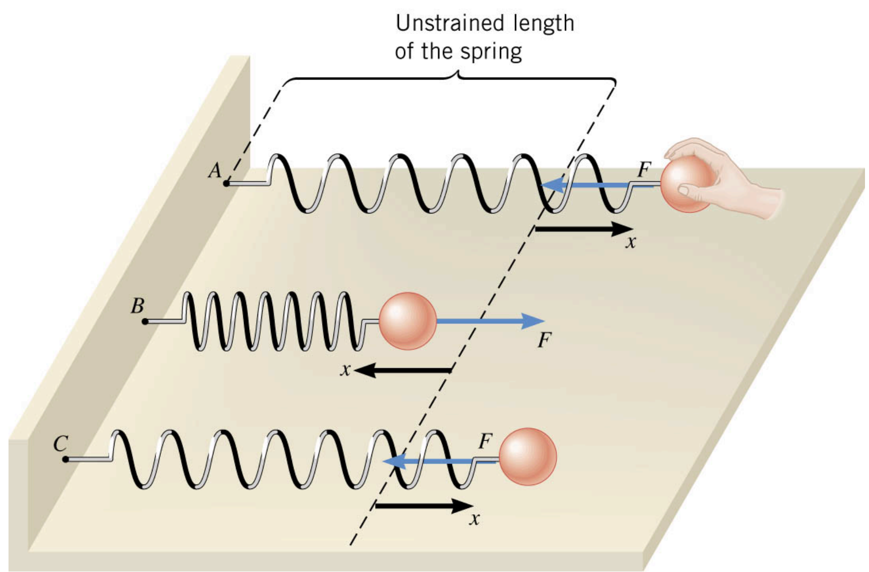
k is the force constant of the spring, in newtons per meter.

When an object attached to a horizontal spring is moved from its equilibrium position and released, a restoring force $F = -kx$ leads to simple harmonic motion. The restoring force has a minus sign because it always points in a direction opposite to the displacement of the spring.

An ideal spring is a spring that behaves according to Hooke’s law.

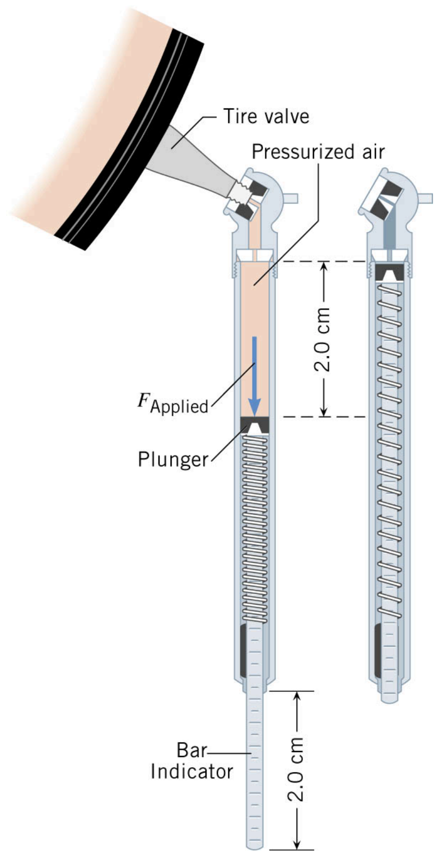
Example 2: *Restoring Force of an Ideal Spring;*

The spring has been stretched, from its unstrained length, for a distance x to the right. The spring exerts a restoring -F force (blue arrow) pointing to the left, opposite to the displacement. The restoring force also leads to a back-and-forth harmonic motion of the object or spring.



Example 3: A Tire Pressure Gauge;

In a tire pressure gauge, the pressurized air from the tire exerts a force F that compresses a spring. The spring constant of the spring is 320 N/m and the bar indicator extends 2.0 cm. What force does the air in the tire apply to the spring?



Data Table		
k	*x*	*F*
320 N/m	0.02 m	?

F = Kx = 320 x 0.02 = 6.4 N

ELASTIC POTENTIAL ENERGY ($PE_{elastic}$)

The energy stored in stretched or compressed spring is called elastic potential energy.

For an ideal spring, the elastic potential energy ($PE_{elastic}$) is given by the formula:

Elastic Potential Energy
$PE_{elastic} = \frac{1}{2} kx^2$

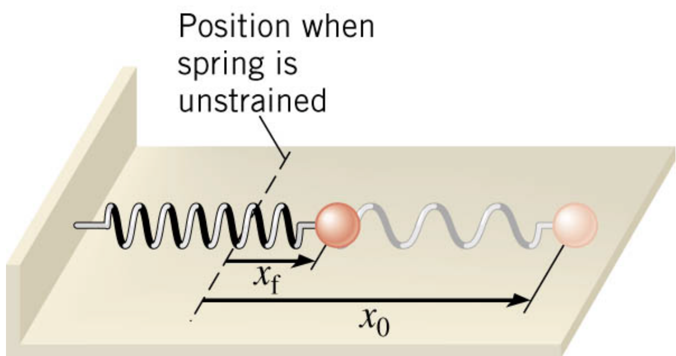
SI Unit of Elastic Potential Energy: joule (J)

ELASTIC POTENTIAL ENERGY and Work

A compressed spring can do work. Using the work-energy theorem, the work $W_{elastic}$ done by the average spring force:

$W_{elastic}$ = Initial potential elastic energy - Final potential elastic energy

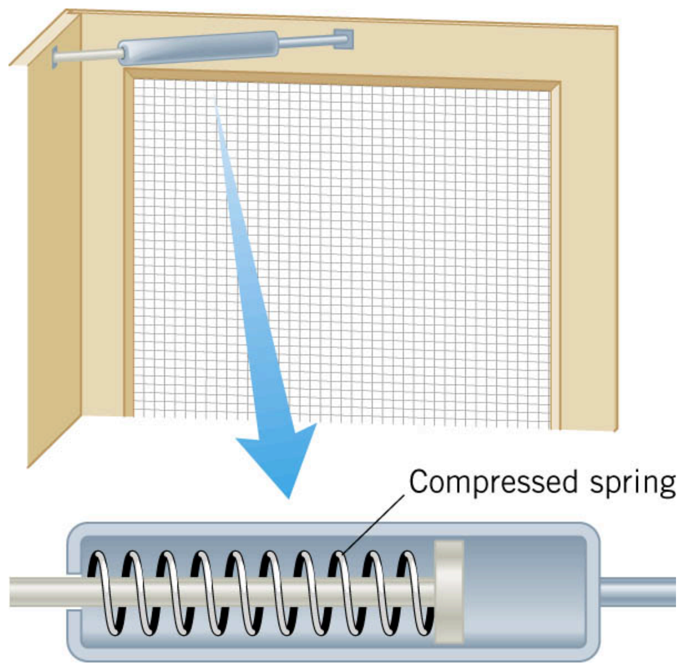
$$W_{elastic} = \frac{1}{2}kx_o^2 - \frac{1}{2}kx_f^2$$



When the object is released, its displacement changes from an initial value of x_o to a final value of x_f. The spring force and the displacement are in the same direction.

Example 4: A Door Closing Unit;

A compressed spring can do work. The elastic potential energy stored in the compresses spring is used to close the door.



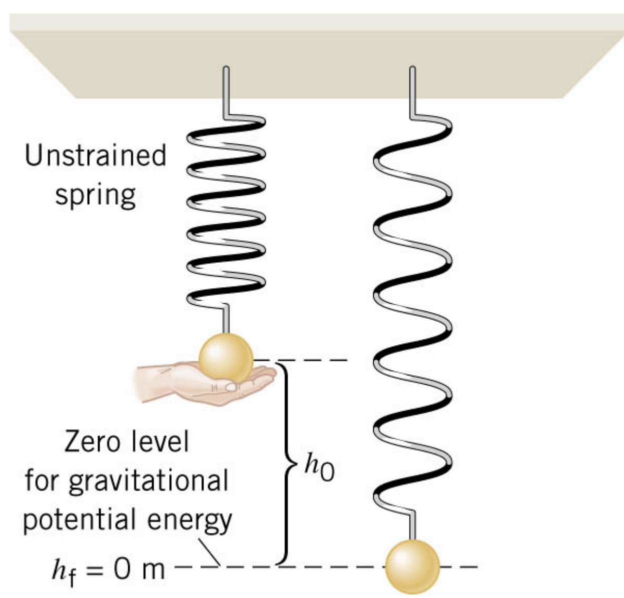
When the door is opened, a spring inside the unit is compressed and has elastic potential energy. When the door is released, the compressed spring expands and does the work of closing the door.

CONSERVATION OF MECHANICAL ENERGY

The total mechanical energy is conserved.

Example 5: *Vertical spring with a mass*

A 0.20-kg ball is attached to a vertical spring. The spring constant is 28 N/m. When released from rest, how far does the ball fall before being brought to a momentary stop by the spring?



Total mechanical energy (E) = Translational kinetic energy (KE) + Gravitational potential energy (PE) + Elastic potential energy ($PE_{elastic}$)

$$E = KE + PE + PE_{elastic}$$

$$E = 1/2\ m\ v^2 + m\ g\ h + 1/2\ k\ x^2$$

The mechanical energy is conserved:

Total final mechanical energy (E_f) = Total initial mechanical energy (E_o)

$$E_f = E_o$$

$$1/2\ m\ v_f^2 + m\ g\ h_f + 1/2\ k\ y_f^2 = 1/2\ m\ v_o^2 + m\ g\ h_o + 1/2\ k\ y_o^2$$

The initial speed $v_o = 0$ m/s. The final speed $v_f = 0$ m/s.

The final height $y_f = 0$. The initial height is y_o .

The spring is unstrained, so $y_o = 0$ m. $h_f = 0$ m.

The displacement: $y_f = - h_o$, the minus sign indicates that the displacement is downward.

so,

$$\frac{1}{2} m v_f^2 + m g h_f + \frac{1}{2} k y_f^2 = \frac{1}{2} m v_o^2 + m g h_o + \frac{1}{2} k y_o^2$$

$$0 + 0 + \frac{1}{2} k y_f^2 = 0 + m g h_o + 0$$

$$\frac{1}{2} k y_f^2 = m g h_o$$

$$\frac{1}{2} k(- h_o)^2 = m g h_o$$

$$h_o = 2 m g / k$$

Data Table			
m	g	k	h_o
0.20 kg	9.8 m/s ^{2}	28 N/m	?

$$h_o = 2 m g / k$$

$$h_o = 2 (0.20) (9.8) / 28$$

$$= 0.14 \text{ m}$$

References:

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The edition was dedicated to the memory of Stella Kupferberg, Director of the Photo Department: "We miss you, Stella, and shall always remember that a well-chosen photograph should speak for itself, without the need for a lengthy explanation"

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