Conservation of Momentum

by

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P3.3b Predict how the change in velocity of a small mass compares to the change in velocity of a large mass when the objects interact (e.g., collide).

P3.5a Apply conservation of momentum to solve simple collision problems.

Items:

1- Conservation of Momentum.

2- Collision and Coupling of Two Objects Moving in the Same Direction.

3- Collision and Coupling of Two Objects Moving in Opposite Directions.

4- Collision and Coupling of Two Objects, One at Rest and One is Moving.

5- Ballistic Pendulum.

PRINCIPLE OF CONSERVATION OF LINEAR MOMENTUM

The law of conservation of momentum is one of the most important laws in science. It can be used to explain what happens when objects collide.

If the net external force (F_{net}) acting on a system of objects is zero, then the total linear momentum (P_{total}) of the system remains unchanged (constant).

$$\vec{F}\,net = 0 \quad \textbf{then} \quad \vec{P}_{total} \text{ is constant}$$

The net external force (F_{net}) is zero when the system of objects is isolated. This means that the objects are not pulled or pushed by anything outside their system. It is okay if they push and pull each other within the system.

Recall: Momentum: $P = m v$

m: mass in kg

v: velocity in m/s

Conservation of Momentum
Total momentum before the collision = Total momentum after the collision
$\vec{p}_{total}$ (**before the collision or impact**) = $\vec{p}_{total}$ (**after the collision or impact**)
Sum of all $(m \times v)$ (**before the collision**) = Sum of all $(m \times v)$ (**after the collision**)

Conservation of linear momentum explains the movement of billiard balls after the collision of the scattering of bowling pins by a bowling ball.

Steps for solving problems using the law of conservation of momentum

In applying conservation of momentum:

a) Define the isolated system, where the net external force is zero.

b) Sketch two pictures: one for the system before the collision and one for the system after the collision.

c) Defines which direction is chosen as the positive direction, then the opposite direction would be the negative direction. The momentum has the same direction of the velocity.

d) write the statement:

Total momentum before the collision = Total momentum after the collision

e) Write it as an equation too. Solve the equation.

Example 1: Assembling a Freight Train; Collision and Coupling of Two Objects Moving in the Same Direction

(a) A loaded railway car of mass (m_1) 6000 kg is rolling to the right at s speed V_{01} 0f 3.0 m/s.

(b) It collides and couples with an empty freight car of mass (m_2) 3000 kg, rolling to the right on the same track at speed V_{02} of 2.0 m/s. What is the speed and direction of the pair after the collision?

(*a*) Before (*b*) After

Data Table					
m_2	m_1	v_{02}	v_{01}	$m_f = m_1 + m_2$	v_f
6000 kg	3000 kg	3.0 m/s	2.0 m/s	9000 kg	?

Steps to solve the problem:

a) The net force acting on the two railway cars is zero. So, the system is isolated.

b) This is a sketch of the collision with the two states: (a) for before the collision and (b) for after the collision.

(*a*) Before (*b*) After

c) The direction to the right is the positive direction. The momentum P has the same direction of the velocity.

Then, V_{01} and V_{02} are positive.

d) When the two railways collide, the momentum (p) is conserved or remains the same.

Total momentum before the collision = Total momentum after the collision

e) Equation:

Sum of all (m x v) (**before the collision**) = Sum of all (m x v) (**after the collision**)

Because the two objects couple together after collision and move with the same speed v_f. The total mass after coupling is the sum of both masses ($m_1 + m_2$):

$$(m_1 \times v_{01}) + (m_2 \times v_{02}) = (m_1 + m_2) \times v_f$$

(**before the collision**) (**after the collision**)

Two Objects Collide and Couple Together , Final Speed Equation of the Pair
$V_f = [(m_1 \times v_{01}) + (m_2 \times v_{02})] / (m_1 + m_2)$

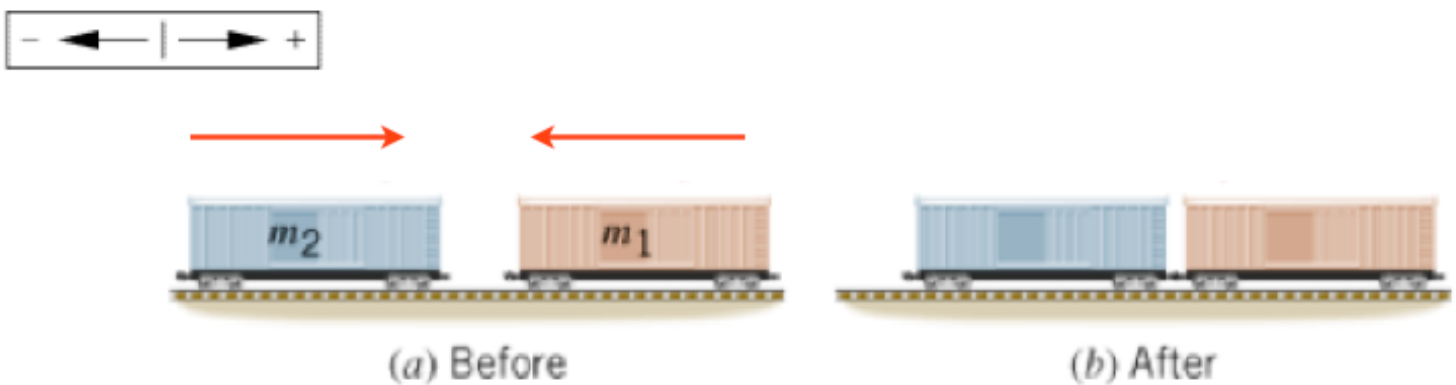
$$V_f = [(3000 \times 2.0) + (6000 \times 3.0)] / 9000$$

$$V_f = 24000/9000 = 2.66 \text{ m/s}$$

So, the pair couple and moved to the right with a speed of 2.66 m/s.

Example 2: *Assembling a Freight Train; Collision and Coupling of Two Objects Moving in Opposite Direction*

A loaded railway car of mass 6000 kg is rolling to the right at 2.0 m/s. Another empty freight car of mass 3000 kg is rolling to the left on the same track at 3.0 m/s. They collide and couple. What is the speed and direction of the pair after the collision?



Data Table					
m_2	m_1	v_{02}	v_{01}	v_f	$m_1 + m_2$
6000 kg	3000 kg	2.0 m/s	**- 3.0 m/s**	?	9000 kg

Steps to solve the problem:

a) The net force acting on the two railway cars is zero. So, the system is isolated.

c) The direction to the right is the positive direction. So, the direction to the left is negative. Then, V_{02} is positive and V_{01} is negative.

d) The momentum (p) is conserved (remains the same).

Total momentum before the collision = Total momentum after the collision

Sum of all (m x v) (**before the collision**) = Sum of all (m x v) (**after the collision**)

e) The two objects collide and couple together:

$(m_1 \times v_{01}) + (m_2 \times v_{02})$ **before the collision**) $= (m_1 + m_2) \times v_f$ (**after the collision**)

$$v_f = [(m_1 \times v_{01}) + (m_2 \times v_{02})] / (m_1 + m_2)$$

$$= [(3000 \times (-3.0) + (6000 \times 2.0)] / 9000$$

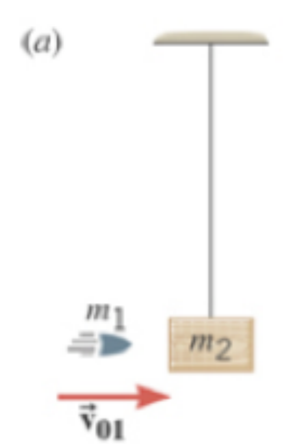
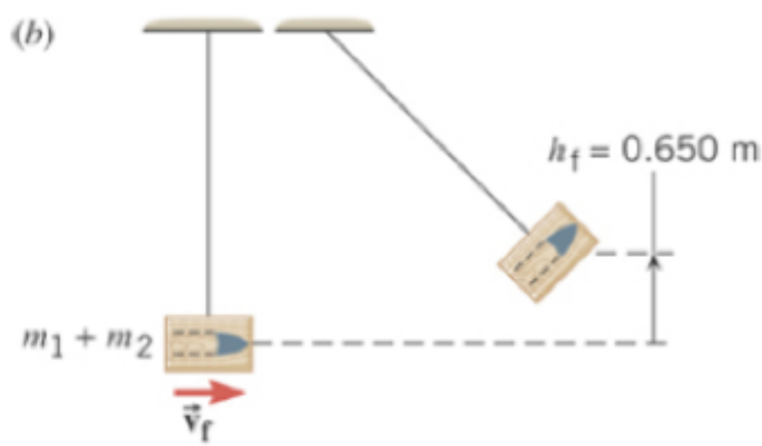
$$= 9000/3000$$

$$= 0.33 \text{ m/s}$$

So, the pair couple and moved to the right with a speed of 0.33 m/s

Example 3: *A Ballistic Pendullum; Collision and Coupling of Two Objects One is Moving and the Other is at Rest.*

(a) A bullet of mass (m_1 = 0.0100 kg) is fired with a speed of (V_{01} = 896 m/s) and collide with a block of wood of mass (m_2 = 2.5 kg) at rest (V_{02} = 0 m/s) and suspended by a wire with negligible mass . After the collision (b), they both swing to a height of h_f. Find the final velocity (V_f) of the wood and the bullet and h_f .

Before Coupling	After Coupling
	

Data Table						
m_2	m_1	v_{02}	v_{01}	$m_{1} + m_{2}$	v_f	h_f
2.5 kg	0.0100 kg	0 m/s	896 m/s	2.51 kg	?	?

a) The net force acting on the bullets and wood is zero. So, the system is isolated.

c) The direction to the right is the positive direction.

d) Total momentum before the collision = Total momentum after the collision

e) Equation: The two objects collide and couple together:

$$v_f = [(m_1 \times v_{01}) + (m_2 \times v_{02})\ ] / (m_1 + m_2)$$

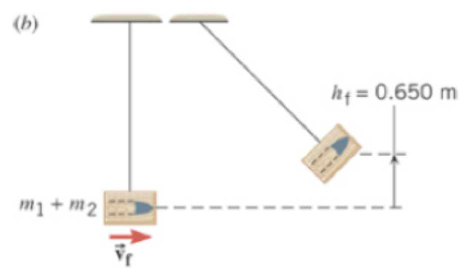
$$= [(0.010 \times 896) + (2.5 \times 0)] / (2.51)$$

$$= 8.96 / 2.51 = 3.569 \text{ m/s} = 3.57 \text{ m/s}$$

So, after the collision, both the block of wood and the bullet move together to the right with a speed of 3.57 m/s.

Calculation of the height h_f.

The system is isolated, the momentum is conserved and the total mechanical energy is conserved too.



The energy is conserved, so:

$$m\, g\, h_f = 1/2\, m\, v_f^2$$

$$(m_1 + m_2)\, g\, h_f = 1/2\, (m_1 + m_2)\, v_f^2$$

We can simplify $(m_1 + m_2)$ and $g = 9.8\ m/s^2$

$$h_f = (1/2\, g)\, v_f^2$$

$$= (1/19.6)\, (3.57)^2 = 0.051 \times 12.75 = 0.650\ m$$

A ballistic pendulum is sometimes used in laboratories to measure the speed of a projectile such as a bullet.

References:

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Physics 1200 Lecture Slides: Dr. Thomas Humanic, Professor of Physics, Ohio State University, *2013-2014 and Current. www.physics.ohio-state.edu/~humanic/*

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The edition was dedicated to the memory of Stella Kupferberg, Director of the Photo Department:"We miss you, Stella, and shall always remember that a well-chosen photograph should speak for itself, without the need for a lengthy explanation"

3) Martindale, D. G. & Heath, R. W. & Konrad, W. W. & Macnaughton, R. R. & Carle, M. A. (1992). *Heath Physics.* Lexington: D.C. Heath and Company

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