Linear Momentum

by

Nada Saab-Ismail, PhD, MAT, MEd, IB

nhsaab.weebly.com

nhsaab2014@gmail.com

P3.3b Predict how the change in velocity of a small mass compares to the change in velocity of a large mass when the objects interact (e.g., collide).

P3.5a Apply conservation of momentum to solve simple collision problems.

Items:

1- Conservation of Momentum.

2- Collision of Two Objects Moving in the Same Direction.

3- From Rest, Two Objects Moving Push off against Each Other.

5- Types of Collisions and Kinetic Energy.

PRINCIPLE OF CONSERVATION OF LINEAR MOMENTUM

The law of conservation of momentum is one of the most important laws in science. It can be used to explain what happens when objects collide.

If the net external force (F_{net}) acting on a system of objects is zero, then the total linear momentum (P_{total}) of the system remains unchanged (constant).

$$\vec{F}\, net = 0 \quad \textbf{then} \quad \vec{P}_{total} \text{ is constant}$$

The net external force (F_{net}) is zero when the system of objects is isolated. This means that the objects are not pulled or pushed by anything outside their system. It is okay if they push and pull each other within the system.

Recall: Momentum: $P = m\,v$

m: mass in kg

v: velocity in m/s

Conservation of Momentum
Total momentum before the collision = Total momentum after the collision
$\vec{p}_{total}$ (**before the collision or impact**) = $\vec{p}_{total}$ (**after the collision or impact**)
Sum of all (m x v) (**before the collision**) = Sum of all (m x v) (**after the collision**)

Conservation of linear momentum explains the movement of billiard balls after the collision of the scattering of bowling pins by a bowling ball.

Steps for solving problems using the law of conservation of momentum

In applying conservation of momentum:

a) Define the isolated system, where the net external force is zero.

b) Sketch two pictures: one for the system before the collision and one for the system after the collision.

c) Defines which direction is chosen as the positive direction, then the opposite direction would be the negative direction. The momentum has the same direction of the velocity.

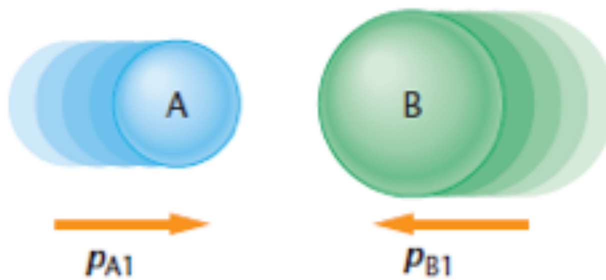
d) write the statement:

Total momentum before the collision = Total momentum after the collision

e) Write it as an equation too. Solve the equation.

Example 1: *Conservation of momentum of billiard balls.*

Two billiard balls A and B are moving toward collision. The momentum of the ball A before the collision is P_{A1}, and the momentum of the ball B before the collision is P_{B1}.



The ball A has a mass m_A and moving with a velocity v_{A1}

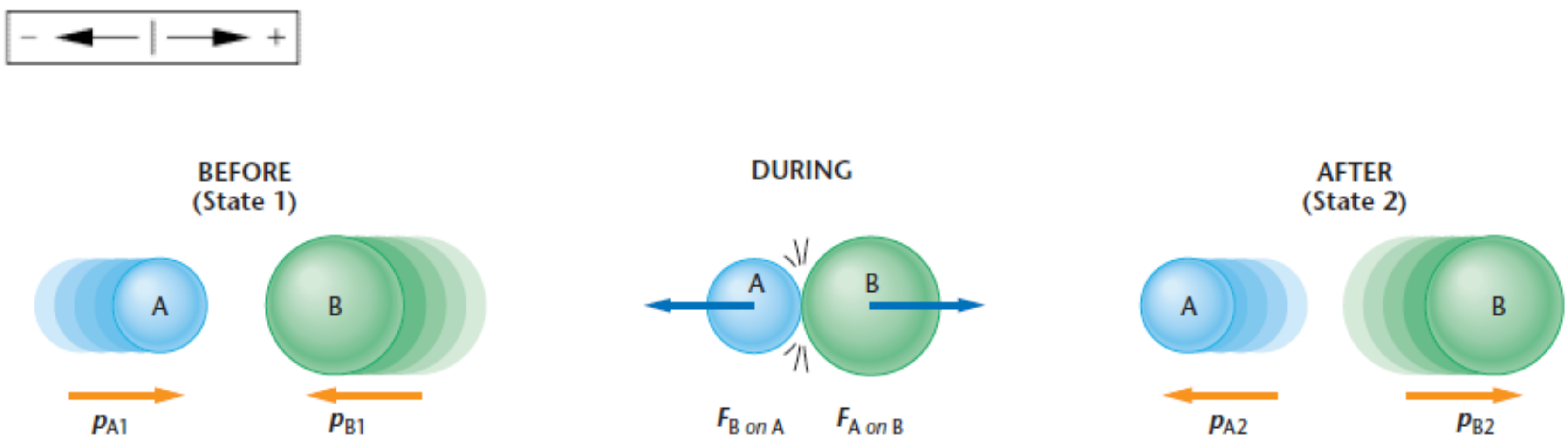
The ball B has a mass m_B and moving with a velocity v_{B1}

Compare the momentum of the balls before and after the collision.

Steps to solve the problem:

a) The net force acting on the two balls above is zero. So, the system is isolated.

b) This is a sketch of the collision with the two states: State 1 for before the collision and State 2 for after the collision.



c) The direction to the right is the positive direction. So, the direction to the left is negative. The momentum P has the same direction of the velocity (V).

Before Collision (State 1):

P_{A1} is positive and P_{B1} is negative.

V_{A1} (velocity of ball A) is positive and V_{B1} (velocity of ball B) is negative.

After Collision (State 2):

P_{A2} is negative and P_{B2} is positive.

V_{A2} (velocity of ball A) is negative and V_{B2} (velocity of ball B) is positive.

d) When the two balls collide, the momentum (p) is conserved or remains the same.

Total momentum before the collision = Total momentum after the collision

e) Equation:

$$P_{A1} + P_{B1} = P_{A2} + P_{B2}$$

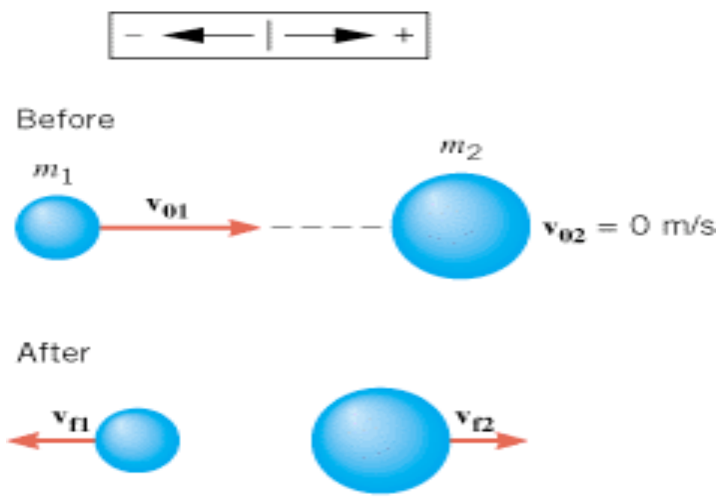
Sum of all (m x v) (**before the collision**) = Sum of all (m x v) (**after the collision**)

$$(m_A \times v_{A1}) + (m_B \times v_{B1}) = (m_A \times v_{A2}) + (m_B \times v_{B2})$$

Example 1: *A Ball Striking Another Ball at Rest.*

Before : A softball of mass m_1 (0.200 kg) moving at a constant velocity v_{01} (8.3 m/s) strikes a second ball of mass m_2 (0.45 kg) that is at rest (v_{02} = 0 m/s).

After the collision, the incoming ball rebounds straight back at a velocity v_{f1} (3.2 m/s) and the second ball moves at a velocity v_{f2}. Calculate v_{f2}.



Data Table					
m_2	m_1	v_{02}	v_{01}	v_{f1}	v_{f2}
0.45 kg	0.200 kg	0 m/s	8.3 m/s	-3.2 m/s	?

a) The net force acting on the two balls is zero. So, the system is isolated.

c) The direction to the right is the positive direction.

So, V_{01} is positive.

V_{f1} is negative and V_{f2} is positive.

d) The momentum (p) is conserved or remains the same.

Total momentum before the collision = Total momentum after the collision

e) Equation:

Sum of all (m x v) (**before the collision**) = Sum of all (m x v) (**after the collision**)

$$(m_1 \times v_{01}) + (m_2 \times v_{02}) = (m_1 \times v_{f1}) + (m_2 \times v_{f2})$$

(**before the collision**) (**after the collision**)

$$(m_1 \times v_{01}) + (m_2 \times v_{02}) - (m_1 \times v_{f1}) = m_2 \times v_{f2}$$

$$[(m_1 \times v_{01}) + (m_2 \times v_{02}) - (m_1 \times v_{f1})] / m_2 = v_{f2}$$

$$[(0.2 \times 8.3) + (0.45 \times 0) - (0.2 \times (-3.2\ m/s))] / 0.45 = v_{f2}$$

$$[1.66 + 0 + 0.64] / 0.45 = v_{f2}$$

$$2.3 / 0.45 = v_{f2}$$

$$v_{f2} = 5.1\ m/s$$

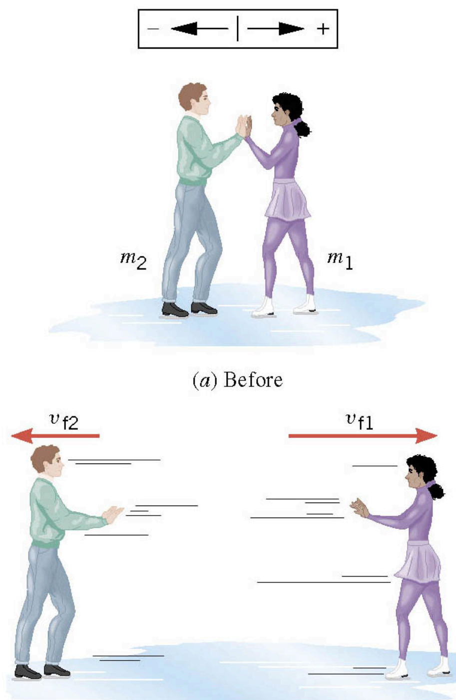
The ball moves with a speed of 5.1 m/s.

Example 2: The law of conservation of momentum is used to explain how fast does the wreckage move immediately after the collision. Conservation of momentum works just as well to describe explosions.

Example 3: Ice Skater;

(a) Before: Starting from rest two skaters push off against each other on ice where friction is negligible. The mass of the woman is m_1 (54 kg) and the mass of the man is m_2 (88 kg).

(b) After: The woman moves away with a velocity $\mathbf{V}_{f1}$ (2.5 m/s) and the man with velocity V_{f2} in the opposite direction. Find the velocity of the man.



(b) After

Data Table					
m_2	m_1	v_{02}	v_{01}	v_{f1}	v_{f2}
88 kg	54 kg	0 m/s	0 m/s	2.5 m/s	?

a) The net force acting on the two balls is zero. So, the system is isolated.

c) The direction to the right is the positive direction.

V_{f2} is negative and V_{f1} is positive.

d) The momentum (p) is conserved.

Total momentum before moving away = Total momentum after moving away

e) Equation:

Sum of all (m x v) (**before**) = Sum of all (m x v) (**after**)

$$(m_1 \times v_{01}) + (m_2 \times v_{02}) = (m_1 \times v_{f1}) + (m_2 \times v_{f2})$$

$$(m_1 \times 0) + (m_2 \times 0) = (54 \times 2.5) + (88 \times v_{f2})$$

$$-(54 \times 2.5) = 88 \times v_{f2}$$

$$v_{f2} = -135 / 88 = -1.5 \text{ m/s}$$

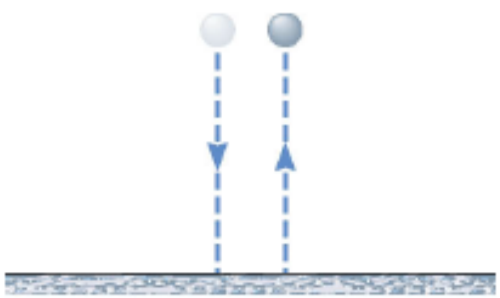
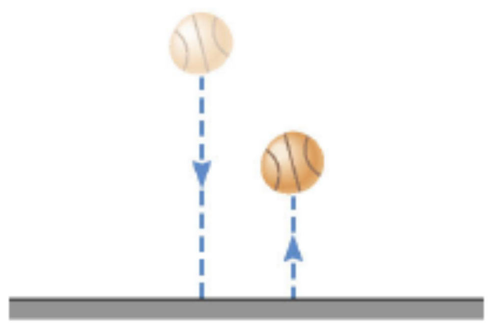
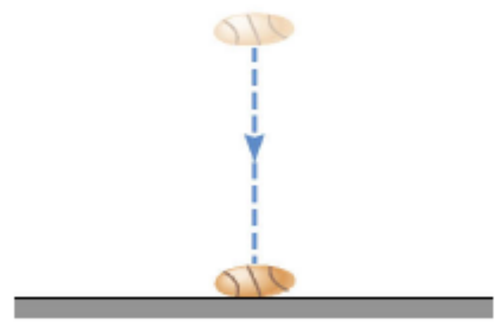
The man moves backward with a speed of 1.5 m/s.

Types of Collisions and Kinetic Energy

Elastic collision -- One in which the total kinetic energy [1/2 (m x v^2)] of the system after the collision is equal to the total kinetic energy before the collision.

Inelastic collision -- One in which the total kinetic energy of the system after the collision is not equal to the total kinetic energy before the collision; if the objects stick together after colliding, the collision is said to be completely inelastic.

In any type of collision, the momentum is always conserved for an isolated system.

Elastic Collision	Inelastic Collision	Completely Inelastic Collision
 (a) Elastic collision	 (b) Inelastic collision	 (c) Completely inelastic collision
1/2 (m x v_i^2) = 1/2 (m x v_f^2)	1/2 (m x v_i^2) # 1/2 (m x v_f^2)	v_f = 0 m/s

v_f ; speed after the collision v_i ; speed before the collision m; mass of the ball

References:

1) Humanic. (2013). www.physics.ohio-state.edu/~humanic/. In Thomas Humanic Brochure Page.

Physics 1200 Lecture Slides: Dr. Thomas Humanic, Professor of Physics, Ohio State University, *2013-2014 and Current*. *www.physics.ohio-state.edu/~humanic/*

2) Cutnell, J. D. & Johnson, K. W. (1998). *Cutnell & Johnson Physics, Fourth Edition*. New York: John Wiley & Sons, Inc.

The edition was dedicated to the memory of Stella Kupferberg, Director of the Photo Department:"We miss you, Stella, and shall always remember that a well-chosen photograph should speak for itself, without the need for a lengthy explanation"

3) Martindale, D. G. & Heath, R. W. & Konrad, W. W. & Macnaughton, R. R. & Carle, M. A. (1992). *Heath Physics.* Lexington: D.C. Heath and Company

4) Zitzewitz, P. W. (1999). *Glencoe Physics Principles and Problems*. New York: McGraw-Hill Companies, Inc.

5) Schnick, W.J. (n.d.). *Calculus-based physics, A Free Physics Textbook*. Retrieved from http://www.anselm.edu/internet/physics/cbphysics/index.html

6) Nada H. Saab (Saab-Ismail), (2009- 2014) Wayne RESA, Bilingual Department, Detroit, Michigan, U.S.A.