Speed of Impact

by

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P4.3B Describe the transformation between potential and kinetic energy in simple mechanical systems (e.g., pendulums, roller coasters, ski lifts).

P4.3f Calculate the impact speed (ignoring air resistance) of an object dropped from a specific height or the maximum height reached by an object (ignoring air resistance), given the initial vertical velocity.

Items:

1- Mechanical Energy.

2- Conservation of Mechanical Energy.

3- Speed of Impact.

MECHANICAL ENERGY

Mechanical energy (E) of an object is the sum of the kinetic energy (KE) and potential energy (PE).

$$E = KE + PE$$

THE PRINCIPLE OF CONSERVATION OF MECHANICAL ENERGY

$$\boxed{E = KE + PE = \text{constant}}$$

Energy can neither be created nor destroyed, but can only be converted from one form to another.

The total mechanical energy of an object remains constant as the object moves, provided that the net work done is zero.

$$E = KE + PE = \text{constant}$$

So,

$$E = 1/2\ m\ v^2 + m\ g\ h = \text{constant}$$

m is the mass of the object, in kilograms (kg)

g is the gravitational field strength. It is a constant value. $g = 9.8\ m/s^2$

h is the height or the vertical displacement the object is moved, in meter (m)

v is the speed of the object, in meters per second (m/s)

KE: kinetic energy is related to the speed (v) of the object: $KE = 1/2\ m\ v^2$

PE: potential energy is related to the height (h) of the object: $PE = m\ g\ h$

The unit of KE and PE is joules (J)

Speed of Impact or Final Speed of a Falling Object

An object moves from an initial position of height (h_o) to a final position of height (h_f). The initial speed is v_o . What is the final speed (v_f) when it reaches the final height h_f ?

The mechanical energy E stays the same in both position.

$$E = KE + PE = \text{Constant}$$

Suppose that $\mathbf{E_o}$ is the initial mechanical energy and $\mathbf{E_f}$ is the final mechanical energy.

The total mechanical energy of the object remains constant as the object moves, because no net work has done on the object.

Total mechanical energy: $E = KE + PE = 1/2\ m\ v^2 + m\ g\ h$

$E_o = E_f$

$$1/2\ m\ v_o{}^2 + m\ g\ h_o = 1/2\ m\ v_f{}^2 + m\ g\ h_f$$

simplify the mass (m) away:

$$1/2\ v_o{}^2 + g\ h_o = 1/2\ v_f{}^2 + g\ h_f$$

$$(1/2\ v_o{}^2 + g\ h_o - g\ h_f) = 1/2\ v_f{}^2$$

$$(1/2\ v_o{}^2 + g\ h_o - g\ h_f) \times 2 = v_f{}^2$$

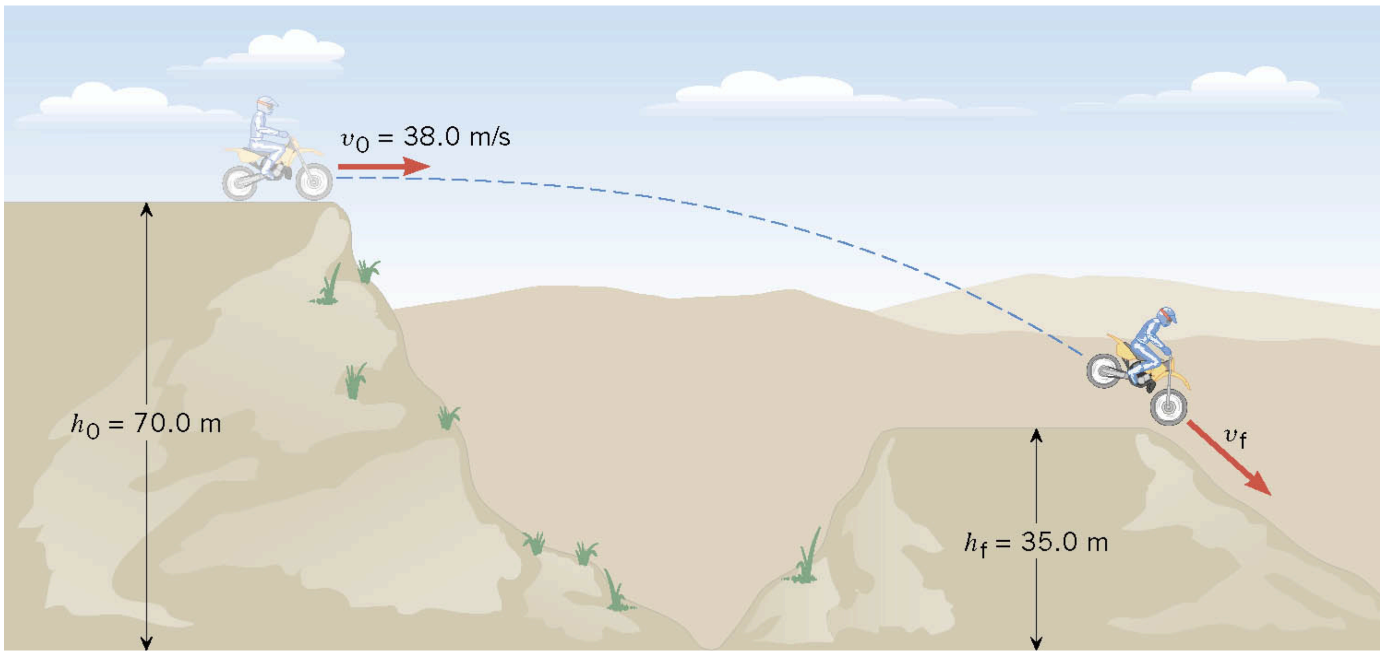
$$v_o{}^2 + 2\ (g\ h_o - g\ h_f) = v_f{}^2$$

$$v_o{}^2 + 2\ g(h_o - h_f) = v_f{}^2$$

Formula for Impact Speed
$v_f = \sqrt{2g\left(h_o - h_f\right) + v_o^2}$

Example 1: A Motorcyclist across the Canyon;

A motorcyclist is trying to leap across the canyon of height 70.0 m. He drives horizontally off the cliff with an initial velocity (v_0) of 38.0 m/s. Ignoring air resistance, find the final speed (v_f) with which the cycle strikes the ground on the other side that has a height of 35.0 m.



Data Table				
h_o	h_f	v_o	v_f	g
70.0 m	35.0 m	38.0 m/s	?	9.8 m/s^2

If an object moves from an initial position (o) to a final position (f), the mechanical energy E stays the same in both position. Energy is conserved

Initial mechanical energy = Final mechanical energy

$$E_o = E_f$$

The final speed of impact can be calculated using the formula:

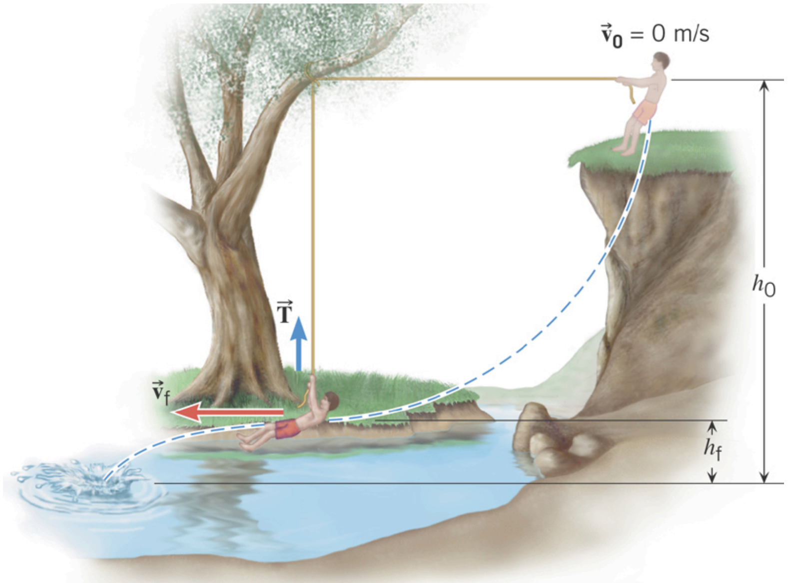
$$v_f = \sqrt{2g(h_o - h_f) + v_o^2}$$

$$v_f = \sqrt{2(9.8\,\mathrm{m/s^2})(35.0\mathrm{m}) + (38.0\,\mathrm{m/s})^2} = 46.2\,\mathrm{m/s}$$

So, the cycle hits the ground on the other side with a speed of 46.2 m/s.

Example 2: *The Favorite Swimming Hole;*

The person starts from rest (v_o = 0 m/s), with the rope held in the horizontal position, swings downward, and then lets go of the rope. Three forces act on him: his weight, the tension in the rope, and the force of air resistance.

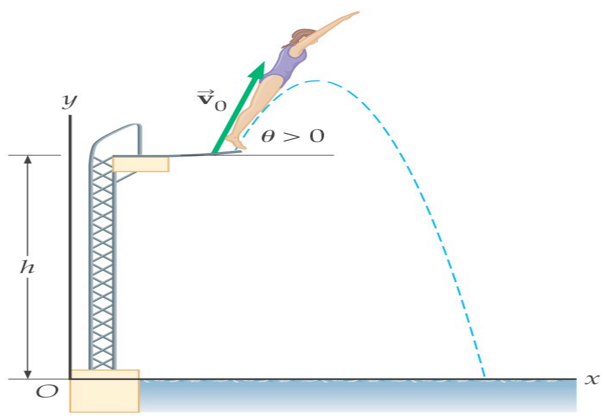


In situations where friction and air resistance are small enough to be ignored and where no other energy is added to the system, the total mechanical energy is conserved. The final speed v_f can be determined using the formula:

$$v_f = \sqrt{2g\left(h_o - h_f\right) + v_o^2}$$

Example 3: Diving from an Edge;

A 56 Kg diver runs and dives from the edge of a board into the water which is located 4.0 m below. She is moving at 8.0 m/s the instant she leaves the board. We are considering the initial state is the moment when she leaves the board. The final state would be the moment she enters the water. Consider the water level as the reference level.



Data Table				
m	h	v_o	g	v_f
56 kg	4.0 m	8.0 m/s	9.8 N/kg	?

Energy is conserved

Initial mechanical energy = Final mechanical energy

$$E_o = E_f$$

$$1/2\ m\ v_o^2 + m\ g\ h_i = 1/2\ m\ v_f^2 + m\ g\ h_f$$

$$1/2\ v_o^2 + g\ h_i = 1/2\ v_f^2 + g\ h_f$$

$$1/2 \times 8^2 + 9.8 \times 4 = 1/2 \times v_f^2 + 9.8 \times 0$$

$$32 + 39.2 = 1/2 \times v_f^2 + 0$$

$$71.2 = 1/2\ v_f^2$$

$$v_f^2 = 71.2 \times 2 = 142.4$$

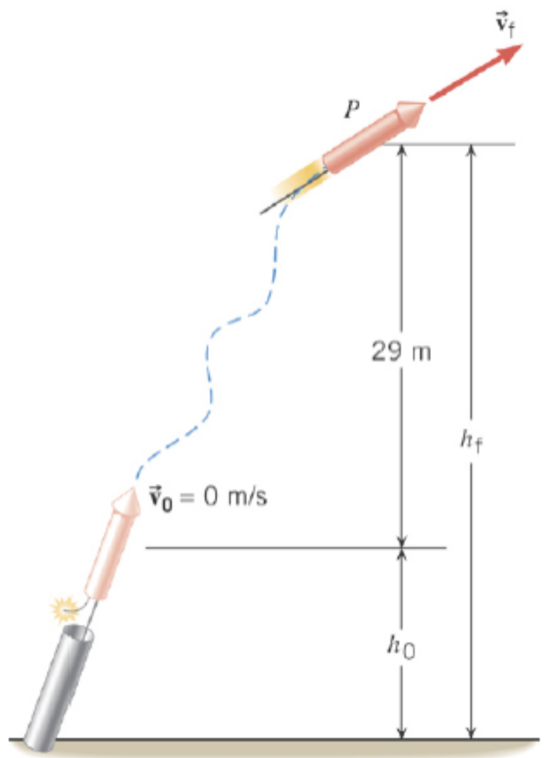
$$v_f = 12 \text{ m/s}$$

We can directly apply the formula:

$$v_f = \sqrt{2g\left(h_o - h_f\right) + v_o^2}$$

$$v_f = 12 \text{ m/s}$$

Example 4: *Transformation of Energy, Fireworks;*



Example 5: *Transformation of Energy, Applications;*

Describe the transformation between potential and kinetic energy in simple mechanical systems (e.g., pendulums, roller coasters, ski lifts).

References:

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The edition was dedicated to the memory of Stella Kupferberg, Director of the Photo Department:"We miss you, Stella, and shall always remember that a well-chosen photograph should speak for itself, without the need for a lengthy explanation"

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