Work Down a Slope

by

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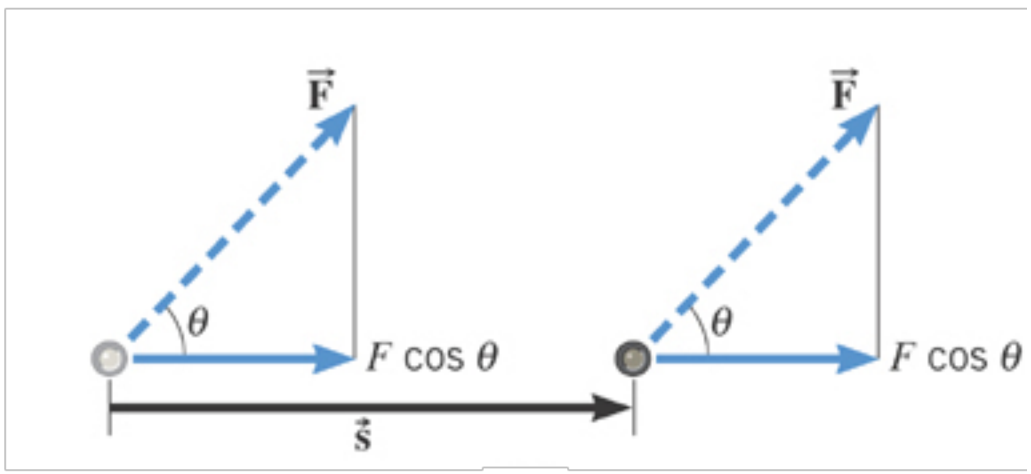
P3.2B Compare work done in different situations.

Items:

1. General Formula of Work.
2. Work Done on an Object by a Constant Force (F), through a Displacement (s), with an Angle (θ).
3. Work Down a Slope.

The general formula of work (W) done on an object by a constant force (F), through a displacement (s), with an angle (θ) between F and s, is:

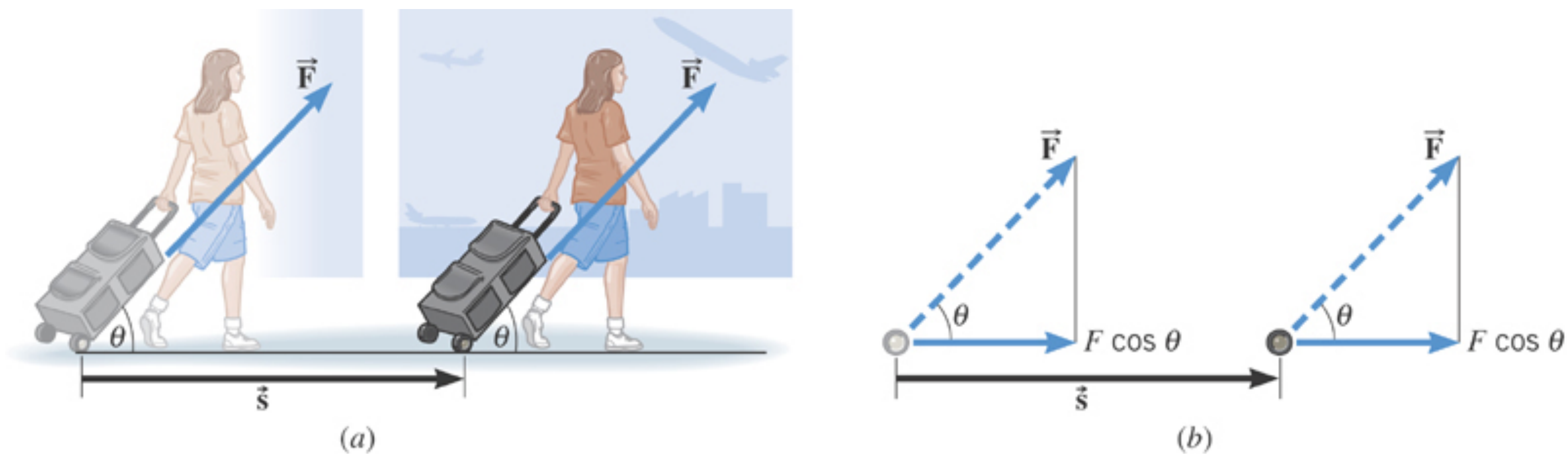
General Formula for Work
$W = (F\cos\theta)s$



Example 1: *Pulling a Suitcase-on-Wheel*

a) A woman is pulling a suitcase-on-wheel with a force F for a distance s and an angle (θ) between F and s.

b) The free body diagram starting with the wheel.



The work done is : $W = (F \cos\theta)s$

Consider that F = 45 N, s = 75 m, $\theta = 50^{o}$ then $\cos 50^{o} = 0.65$,

$$W = (F \cos\theta)s = 45 \times 0.65 \times 75 = 2170 \text{ J.}$$

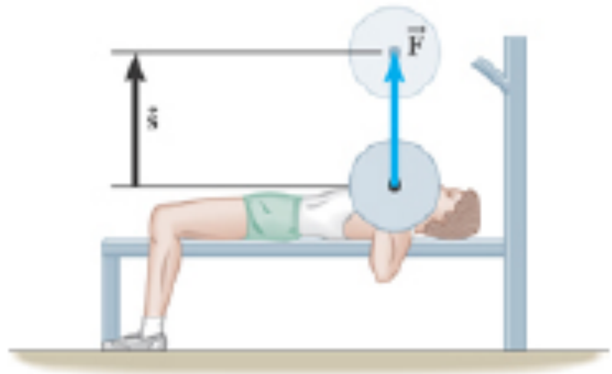
Cases when Angle θ is equal to 0^o or 180^o

Work can be either positive or negative, depending on whether the force and displacement are in the same or opposite direction.

Example 2: In the bench press, work is done during both the lifting and lowering phase of the barbell's motion (a).

(a)

(b) Positive Work ($\theta = 0^{o}$):



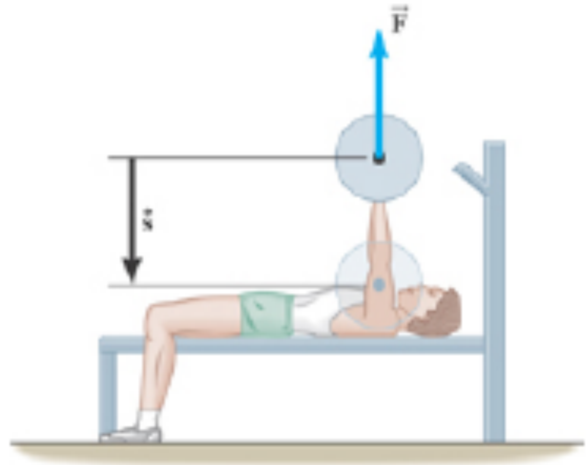
(b)

The weight lifter raises the barbell a distance above his chest. Therefore, S and F are parallel and in the same direction. So, the angle θ between and S and F is 0^{o}. We know that $\cos 0^{o} = 1$

$$W = (F\cos\theta)s = W = (F\cos 0^{o})s = Fs.$$

During the lifting phase, the force F does a positive work on the barbell.

(c) Negative Work ($\theta = 180^{o}$):



(c)

The weight lifter lowers the barbell the same distance. Therefore, S and F are parallel and in opposite direction. So, the angle θ between and S and F is 180^{o}. We know that $\cos 180^{o} = -1$

$$W = (F \cos\theta)s = W = (F \cos 180^{o})s = - F \times s.$$

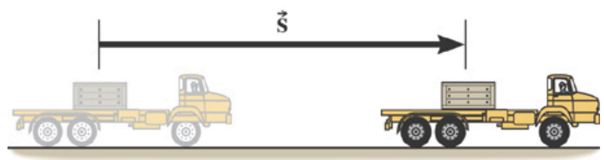
During the lowering phase, the force F does a negative work on the barbell.

Cases when angle θ is equal to 90^o

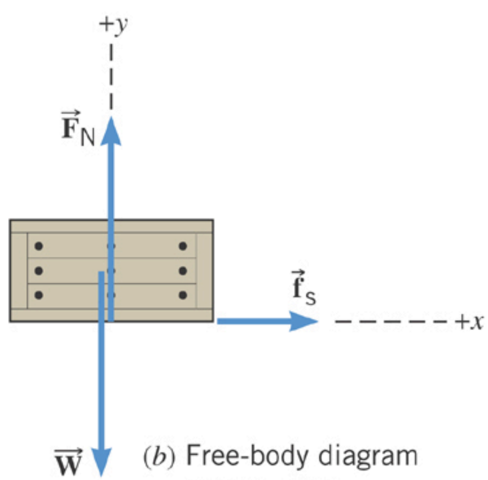
Example 3: *Accelerating a Crate*:

a) The truck is accelerating for a displacement of s. The crate does not slip.

b) The free-body diagram of the forces on the crate: normal force (F_N), weight (W), friction force (f_s).

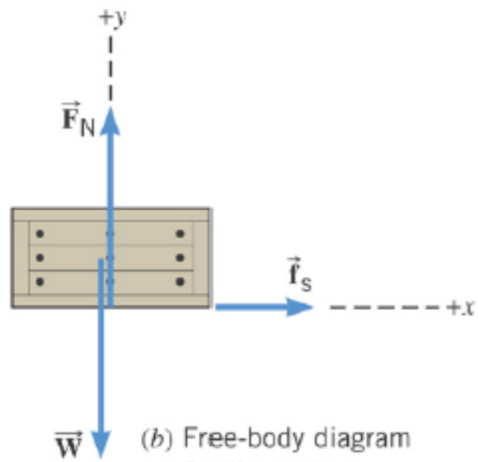


(a)



(b) Free-body diagram for the crate

Analysis of the work done by the normal force (F_N) and weight (W), ($\theta = 90^o$):



(b) Free-body diagram for the crate

Both F_N and W are perpendicular to the direction of the displacement.

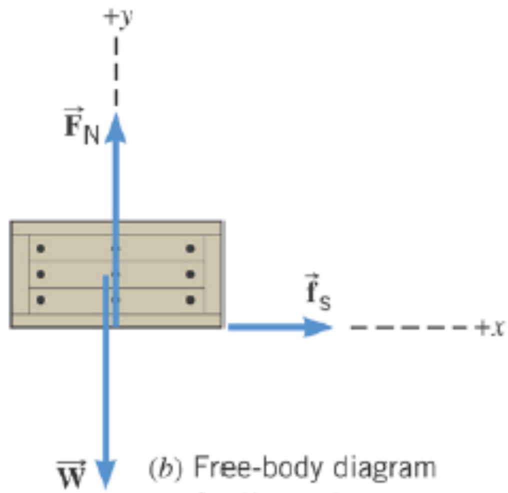
The angle between the displacement (s) and F_N is 90^o . The angle between the displacement (s) and W is also 90 degrees. We know that cos 90^o = 0,

$$W = (F \cos\theta)s = W = (F \cos 90^o)s = 0.$$

So, for both F_N and W, the amount of work is zero. Both forces cancel each other.

> The work is zero if the force is perpendicular to the displacement ($\theta = 90^o$)

Analysis of the work done by the friction force (f_s) ($\theta = 0^o$)



(b) Free-body diagram for the crate

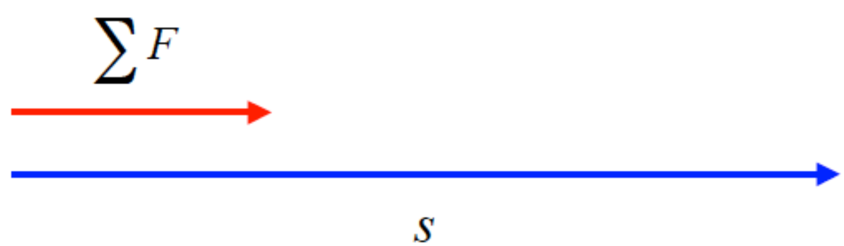
The friction force (f_s) and the displacement (S) are parallel and in the same direction. So, the angle θ between and S and F is 0^o. We know that cos 0^o = 1.

$$W = (f_s \cos\theta)s = W = (f_s \cos 0^o)s = f_s s = (m \times a) s$$

m is mass of the crate
a is acceleration of the truck.

General formula for work when the net force and displacement are in the same direction.

Consider a constant net external force acting on an object. The object is displaced a distance **s**, in the same direction as the net force.



The work can be given by the formula

$$W = \left(\sum F\right)s = (ma)s$$

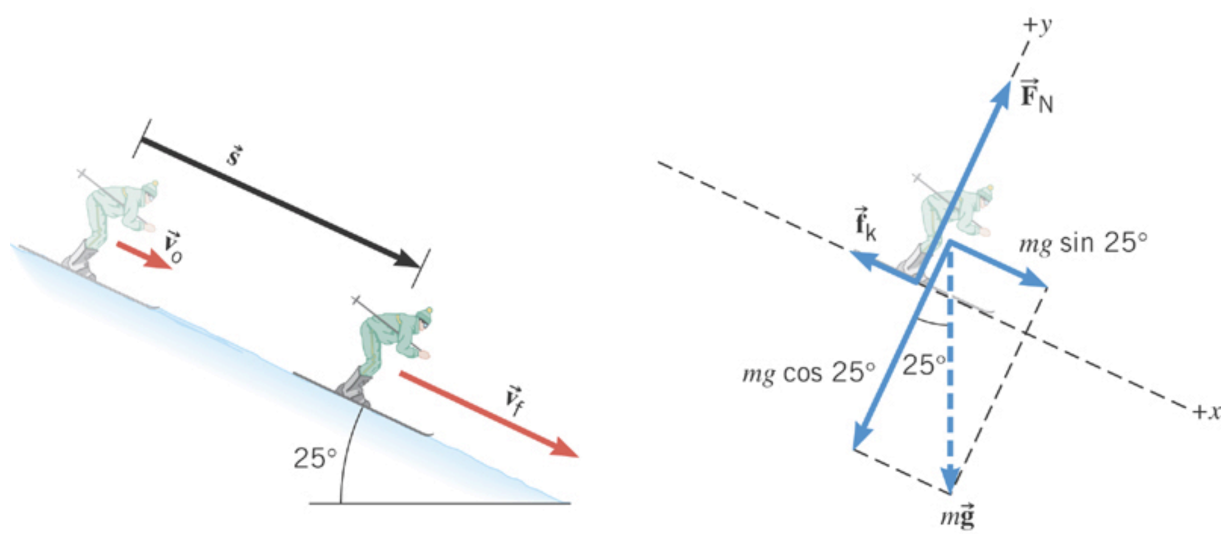
$\sum F$ is the net force or sum of all the forces acting on the object.

Work Down a Slope

Example 4: A skier down a slope; A 58 kg skier is coasting down a slope with an angle 25^o. She accelerates down the slope because of the gravitational force. The kinetic frictional force (f_k) of 71 N opposes her motion. Ignoring air resistance, determine the work done at a displacement point (s) of 57 m downhill.

a) The skier moving along the displacement (s). V_0 is the initial speed. V_f is the final speed.

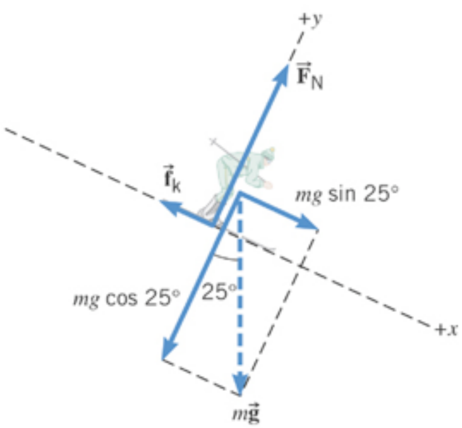
b) The free body diagram for the skier.



(a)

(b) Free-body diagram for the skier

Analysis of the free-body diagram



(b) Free-body diagram for the skier

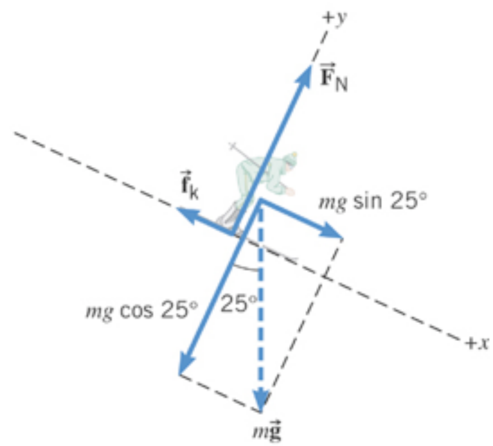
Forces along the y axis

The weight of the skier mg components along the y axis is m g cos 25^o**.**

There is also the normal force (F_N) of equal magnitude but opposite direction:

FN = - mg cos 25^o.

Sum of the forces along the y axis is 0.



(b) Free-body diagram for the skier

Forces along the x axis

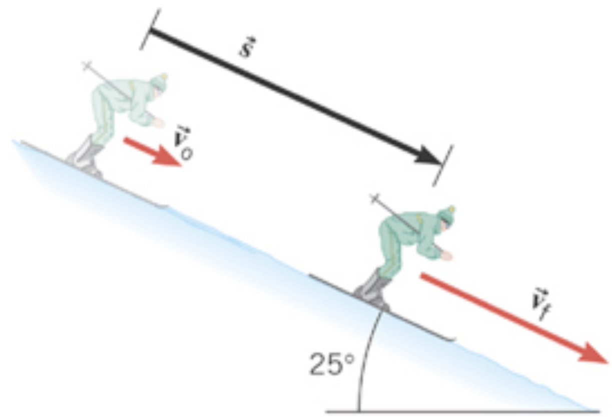
The weight of the skier mg components along the x axis is m g sin 25^{o}**.**

There is also the kinetic frictional force (f_k) of 71 N opposing the motion.

Sum of the forces along the x axis = (m g sin 25^{o}) - 71 =

(58 x 9.8 x 0.42) - 71 =

170 N



(a)

The displacement and net force are in the same direction (angle 0^{o})

W = net force x displacement = 170 x 57 = 9700 J

Example 5:

References:

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Physics 1200 Lecture Slides: Dr. Thomas Humanic, Professor of Physics, Ohio State University, *2013-2014 and Current*. *www.physics.ohio-state.edu/~humanic/*

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The edition was dedicated to the memory of Stella Kupferberg, Director of the Photo Department:"We miss you, Stella, and shall always remember that a well-chosen photograph should speak for itself, without the need for a lengthy explanation"

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