Math or Trigonometry for Two Dimensional Motion and Vectors

by

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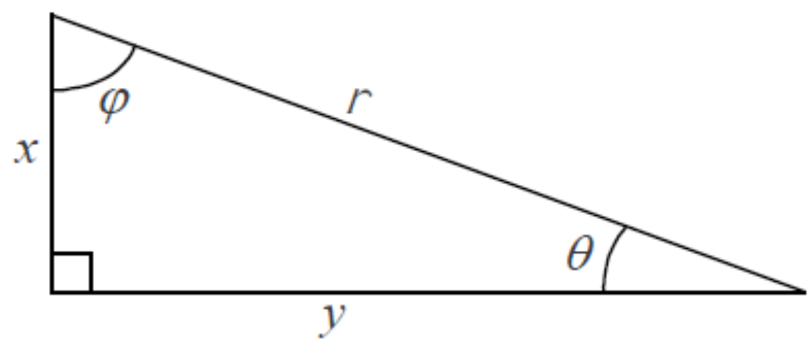
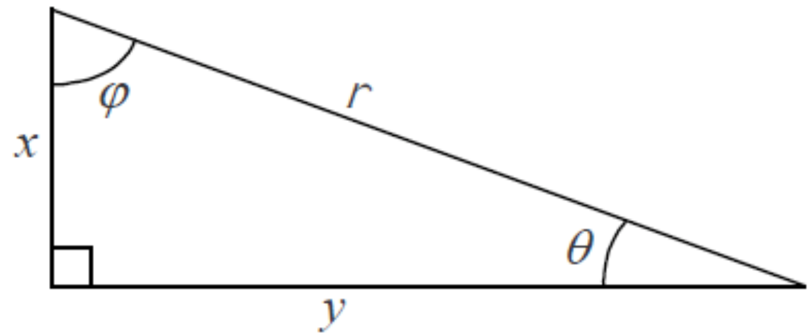
Pythagorean Theorem for Right Triangles

To be able to solve problems related to two dimensional motion and vectors, we need to use the Pythagorean Theorem for right triangles and the relationship between the sides and the angle.



(Length of the hypotenuse)^2 = (Length of one leg)^2 + (Length of the other leg)^2



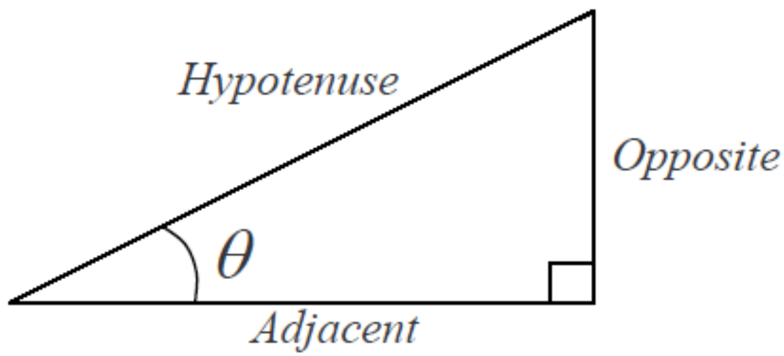
Pythagorean Theorem → $r^2 = x^2 + y^2$

Subtract x^2 from both sides of the equation → $r^2 - x^2 = y^2$

Swap sides → $y^2 = r^2 - x^2$

Take the square root of both sides of the equation → $y = \sqrt{r^2 - x^2}$

Relations Between the sides and the angles of a right triangle.



The relations between the sides and the angles of a right triangle are given below. A convenient mnemonic for doing so is **"SOHCAHTOA"** pronounced as a single word.

S = Sin

O = Opposite

H = Hypotenuse

C = Cos

T = Tangent

“SOHCAHTOA”

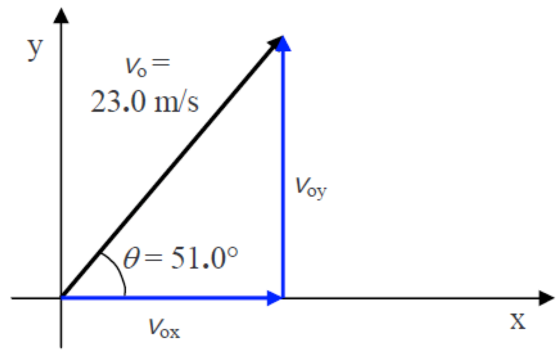
$$\text{SOH reminds us that: } \sin\theta = \frac{Opposite}{Hypotenuse}$$

$$\text{CAH reminds us that: } \cos\theta = \frac{Adjacent}{Hypotenuse}$$

$$\text{TOA reminds us that: } \tan\theta = \frac{Opposite}{Adjacent}$$

Example 1:

Find the component velocities of a rocket traveling at 23.0 m/s at an angle of 51° to the ground.



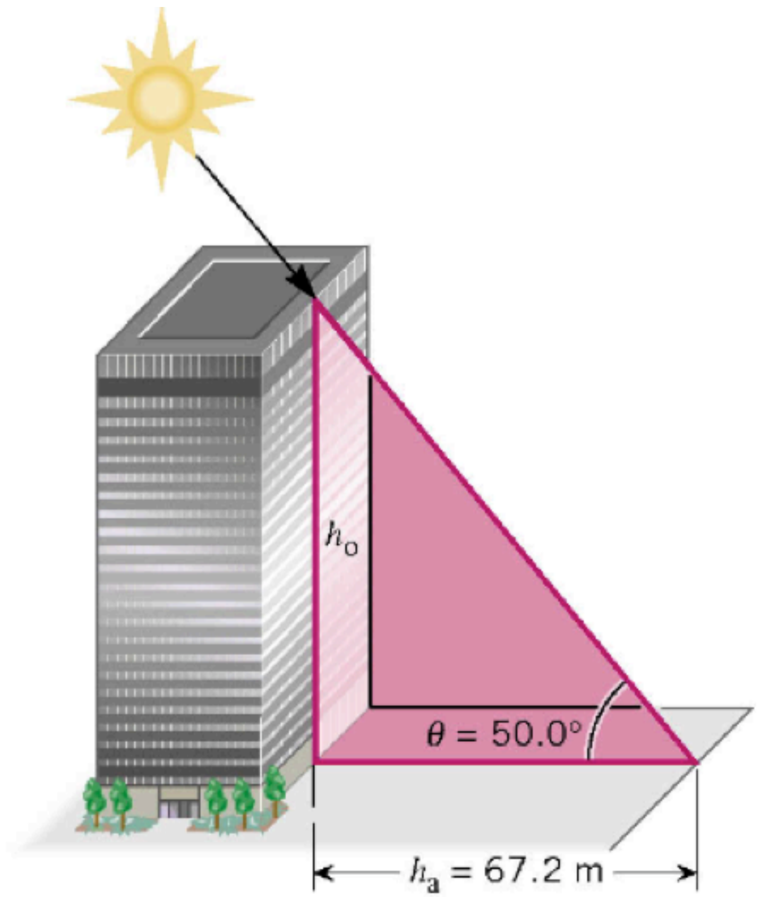
$\cos\theta = \frac{v_{ox}}{v_o}$ $v_{ox} = v_o \cos\theta$ $v_{ox} = 23.0\frac{m}{s}\cos 51.0°$ $v_{ox} = 14.47\frac{m}{s}$	$\sin\theta = \frac{v_{oy}}{v_o}$ $v_{oy} = v_o \sin\theta$ $v_{oy} = 23.0\frac{m}{s}\sin 51.0°$ $v_{oy} = 17.87\frac{m}{s}$

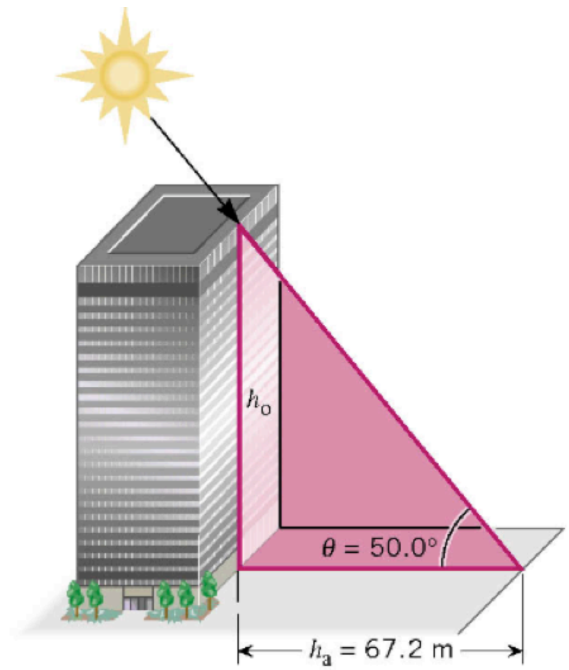
(Length of the hypotenuse)^2 = (Length of one leg)^2 + (Length of the other leg)^2

$$(23)^2 = (14.47)^2 + (17.87)^2$$

Example 1: *Height of a Building;*

A building of height (h_o) casts a shadow (h_a) of 67.2 m when the angle (θ) of the Sun's rays with respect to the ground is 50.0°. What is the height (h_o) of the building?





$\tan\theta$ = opposite/adjacent

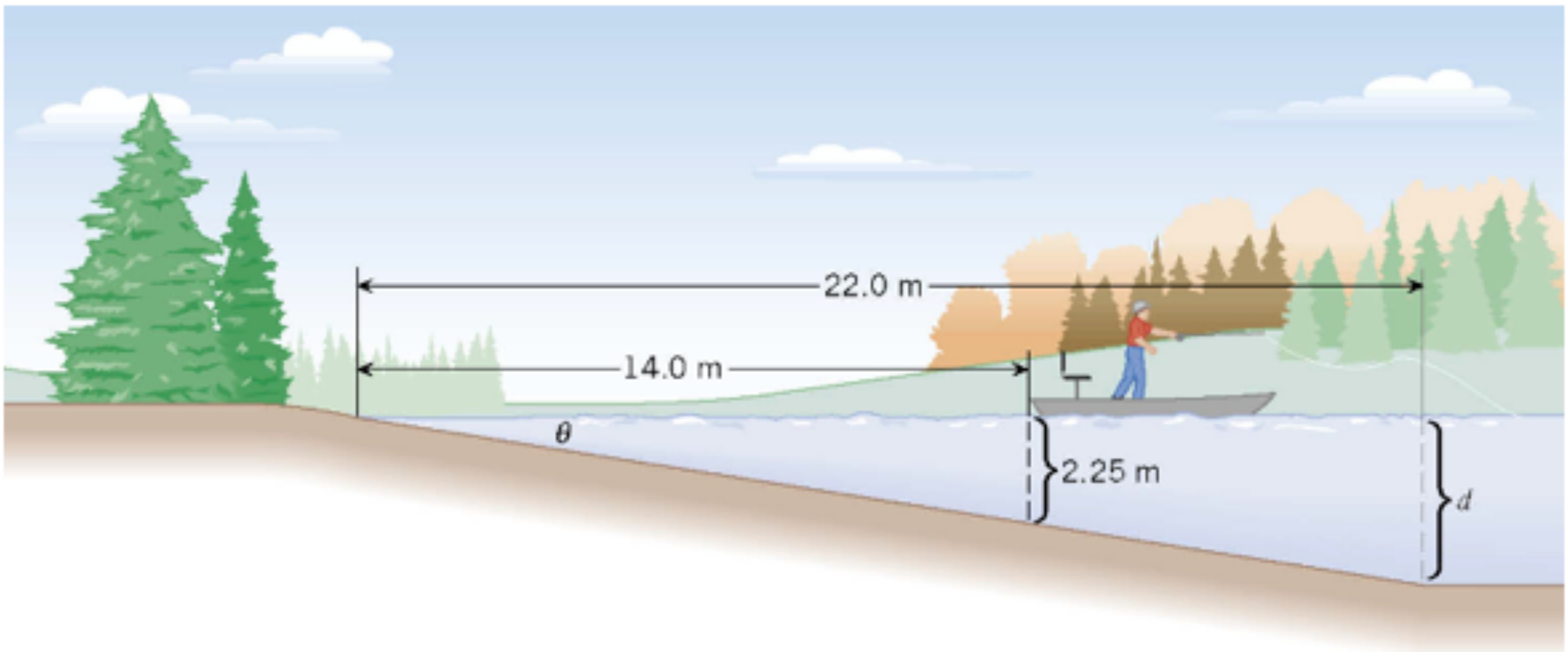
$\tan 50^o = h_o / h_a$

$h_o = h_a \times \tan 50 = 67.2 \times \tan 50^o = 80.0$ m

So, the building is 80.0 m tall.

Example 3: *Lakefront Drop.*

At what angle does the lakefront drop off if the depth of the lake at a distance of 14.0 m is 2.25 m?



$$\tan\theta = \text{opposite/adjacent}$$

$$\theta = \tan^{-1}(\text{opposite/adjacent})$$

$$= \tan^{-1}(2.25\text{ m} / 14\text{ m}) = 9.13^{o}$$

References:

1) Humanic. (2013). www.physics.ohio-state.edu/~humanic/. In Thomas Humanic Brochure Page.

Physics 1200 Lecture Slides: Dr. Thomas Humanic, Professor of Physics, Ohio State University, *2013-2014 and Current. www.physics.ohio-state.edu/~humanic/*

2) Cutnell, J. D. & Johnson, K. W. (1998). *Cutnell & Johnson Physics, Fourth Edition*. New York: John Wiley & Sons, Inc.

The edition was dedicated to the memory of Stella Kupferberg, Director of the Photo Department: "We miss you, Stella, and shall always remember that a well-chosen photograph should speak for itself, without the need for a lengthy explanation"

3) Schnick, W.J. (n.d.). *Calculus-based physics, A Free Physics Textbook*. Retrieved from http://www.anselm.edu/internet/physics/cbphysics/index.html

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