Uniform Circular Motion

by

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P2.2D State that uniform circular motion involves acceleration without a change in speed.

P2.2f Describe the relationship between changes in position, velocity, and acceleration during periodic motion.

P3.4D Identify the force(s) acting on objects moving with uniform circular motion (e.g., a car on a circular track, satellites in orbit).

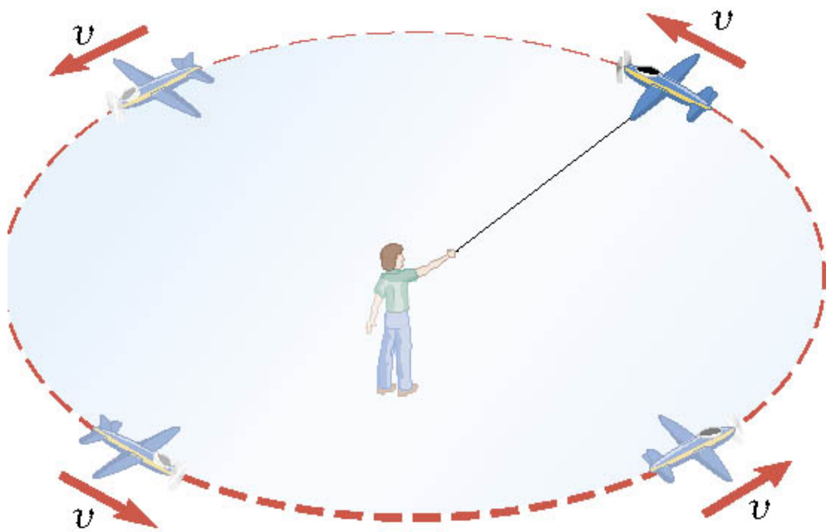
Items:

1. Uniform Circular Motion, Speed, Acceleration.
2. Speed of uniform circular motion.
3. Centripetal Force of Uniform Circular Motion,

5. Satellites in Orbits.

Uniform Circular Motion

Uniform circular motion is the motion of an object traveling on a circular path at a constant speed. So, the speed is constant, but the direction of the velocity vector (v) keeps changing all around. Therefore, the object is accelerating.



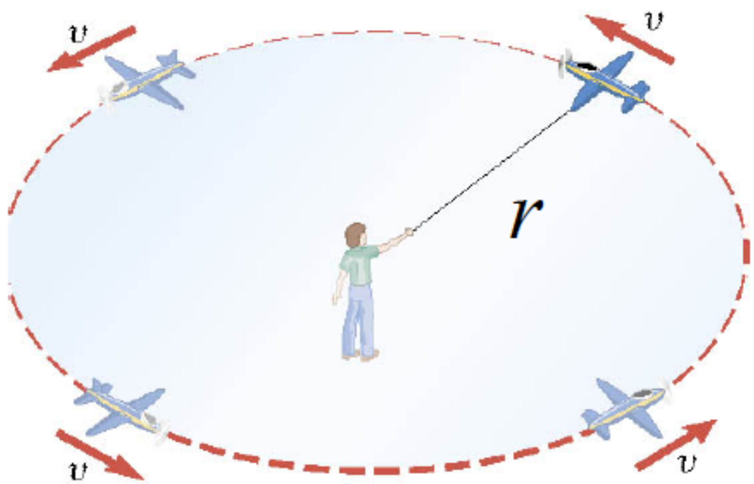
The motion of this model airplane moving at a constant speed but changing the direction of velocity vector is an example of uniform circular motion.

Characteristics of Uniform Circular Motion

Uniform circular motion has variable:

1. The velocity vector (v)
2. The period T: It is the time required to travel only once around the circle. It is also called the period of revolution or simply the period of the circular motion.
3. The radius of the circle: r

The circumference of the circle = $2\,\pi\,r$ with ($\pi = 3.14$)



Speed of Uniform Circular Motion (v)

There is a relationship between period and speed. The speed of a uniform circular motion is the length of one circle divided by the time it takes to complete one circle.

The length of one complete circle is the circumference = $2\pi r$ ($\pi = 3.14$)

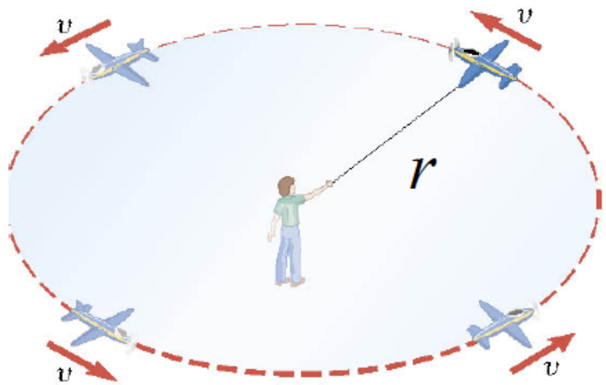
The time to complete one circle is the period (T)

Speed = circumference/ period

Centripetal Speed Formula	
Centripetal Speed (v)	$v = 2\pi r / T$

Example 1: *A model airplane in circular motion:*

Suppose that the model airplane is moving in a circular motion with a radius of 1.2 m. The time to complete one circle or period is 2 s. Calculate the speed (v) of the model airplane.



Data Table			
T	r	Π	v
2 s	0.29 m	3.14	?

$$v = 2\pi r / T$$

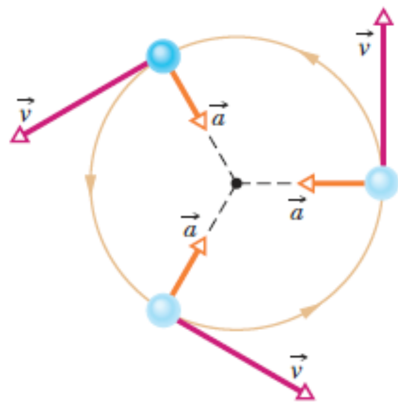
$$= 2\,(3.14)\,1.2 / 2$$

$$= 3.77 \text{ m/s}$$

So, the speed is 3.77 m/s, tangent to the circle and in the direction of the motion

Acceleration of Uniform Circular Motion
Centripetal Acceleration "Center seeking" (a_c)

An object moving at a uniform speed (v) in a circle has an acceleration towards the center of the circle. This is called a centripetal acceleration (*a or* a_c *).*



The velocity ($\vec{v}$) is always directed tangent (touching the edge) to the circle in the direction of the motion.

The acceleration (*a or* a_c *)* is always directed radially inward toward the center and continually changes direction as the object moves.

Centripetal Acceleration (a) Formula	
Centripetal Acceleration (a)	$a = \frac{v^2}{r}$
Another Equation Centripetal Acceleration (a)	$a = \frac{v^2}{r}$ $= \frac{(2\,\Pi\, r/T)^2}{r} = \frac{4\,\Pi^2 r^2}{r\,T^2}$ $a = \frac{4\,\Pi^2 r}{T^2}$

T: is the period of the motion. It is the time needed to complete one circle.

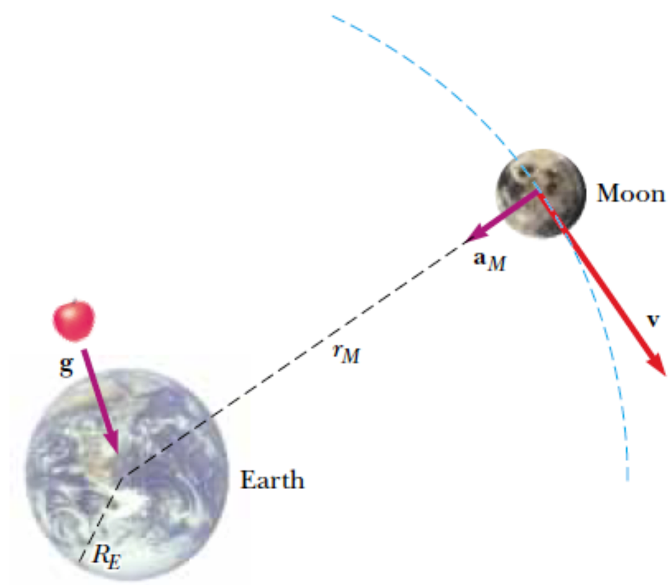
r: is the radius of the circle.

$\Pi = 3.14$

Example 2: *The moon revolving around the Earth.*

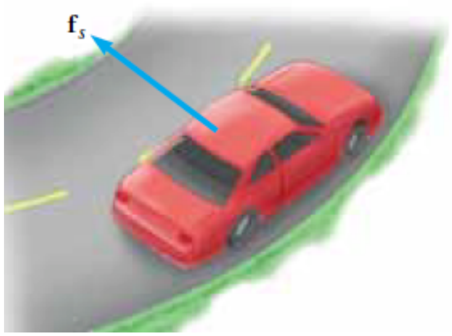
It is moving at a uniform speed (v) in a circle around the Earth and has an acceleration (a_M) towards the center of the Earth . Notice The velocity (v) is always directed tangent (touching the edge) to the circle in the direction of the motion.

Earth is surrounded by a gravitational force field $g = 9.8\ \mathrm{m/s^2}$ that pulls every mass towards its center as shown by the action on the apple.



Example 3: *A Car with Circular Curve:*

A car turns a circular curve with a speed (v) of 20 m/s. If the radius (r) of the curve is 100 m, what is the centripetal acceleration (a) of the car?



Data Table		
a_c	r	v
?	100 m	20 m/s

According to the formula of Centripetal Acceleration:

$$a = \frac{v^2}{r}$$

$$= (20)^2 / 100 = 4.0 \text{ m/s}^2$$

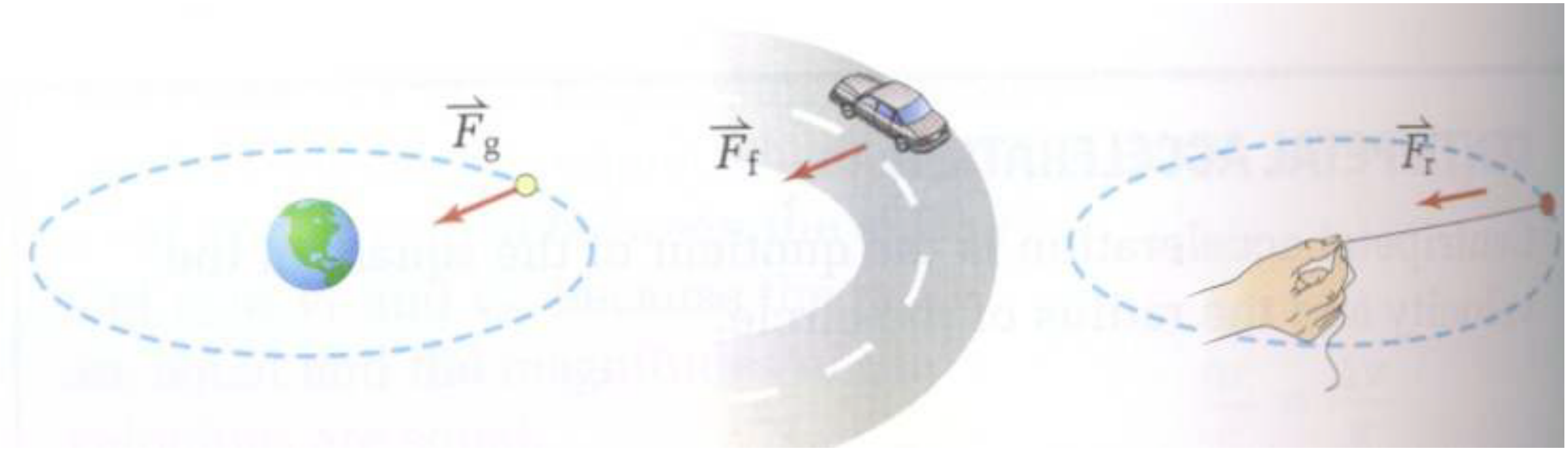
Force of Uniform Circular Motion
Centripetal Force (F_c)

The centripetal force (F_c) keeps the object moving in a circular path.

The direction of the centripetal force always points toward the center of the circle and continually changes direction as the object moves.

Centripetal force comes in many forms.

Examples are shown in the picture below:



In the first picture, gravity provides centripetal force (F_g) that keeps the moon in a circular path around the Earth.

In the second picture, static friction provides centripetal force (F_f) that keeps the car in a circular path.

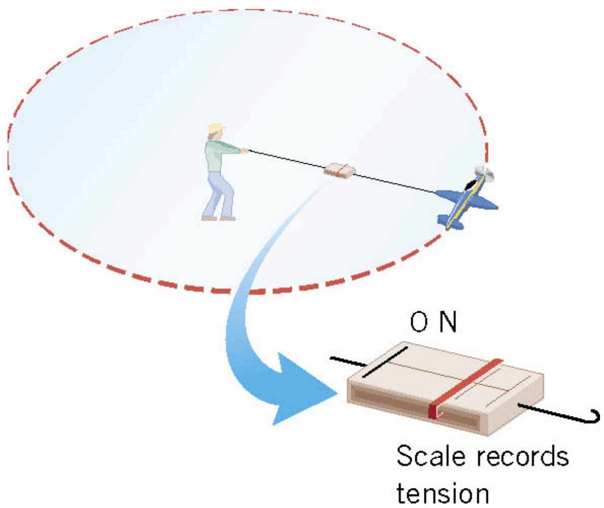
In the third picture, the tension of the rope provides centripetal force (F_r) that keeps the ball moving in a circular path.

According to Newton's second law of motion: Net Force = Mass x Acceleration

Centripetal Force Formula	
Centripetal Force (F_C)	$F_C = m\,a$ $= m \dfrac{v^2}{r}$
Another Equation Centripetal Force (F_C) *T is the period*	$F_C = m\,a$ $= m \dfrac{v^2}{r}$ $= m \dfrac{4\,\Pi^2 r}{T^2}$

Example 4: *A model airplane*

A model airplane of mass (m) 0.90 kg is moving in a circular path with a constant speed (v) of 19 m/s. What is the Tension (T) or centripetal force (F_c) if the radius (r) is 17 m?



Data Table			
$T = F_C$	r	m	v
?	17 m	0.90 kg	19 m/s

$$F_c = T = m\,a$$

$$= m\,\frac{v^2}{r}$$

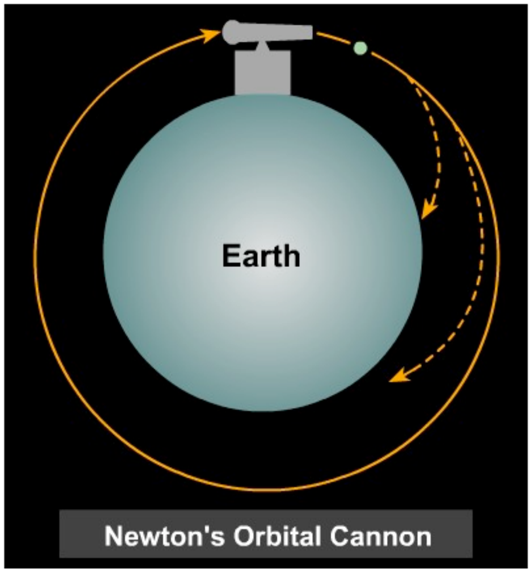
$$= 0.90 \times (19)^2 / 17$$

$$= 19\ \text{N}$$

So the tension or centripetal force is 19 N.

Satellites in orbits and projectiles;

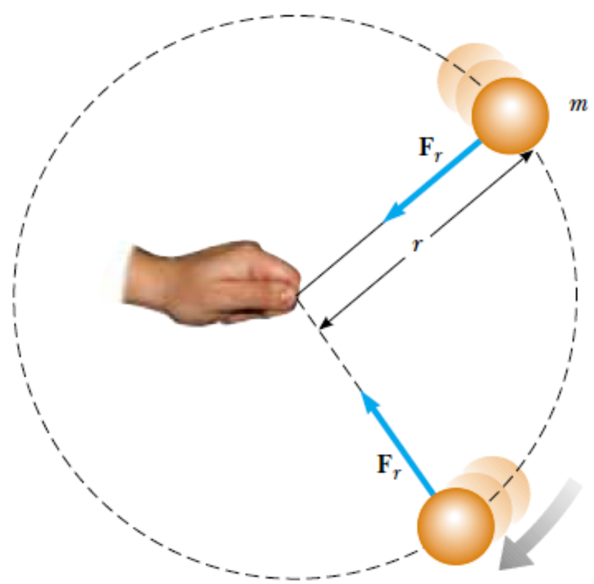
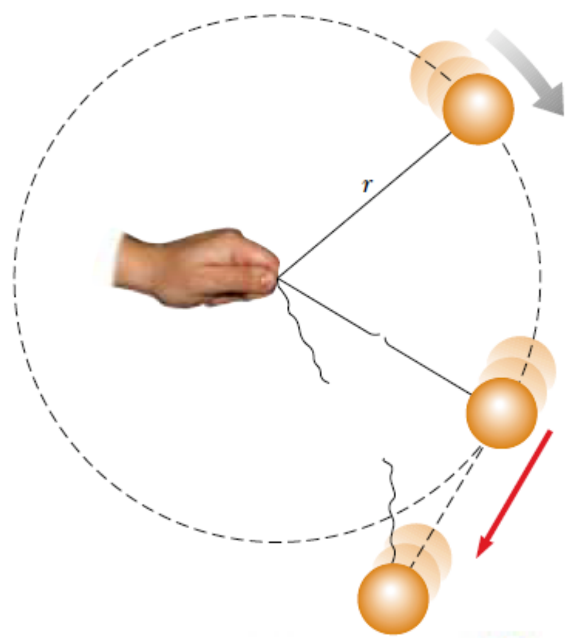
It was Newton who first saw the connection between falling objects, projectiles, and satellites in orbits. He gave this example:



If a projectile is fired at 8 km/s (18000 miles/h), it will fall 5 m every 1 second. The cannon ball's path curves downwards at the same rate as the Earth's curvature. The cannon ball is then in orbit, falling towards the Earth but never landing.

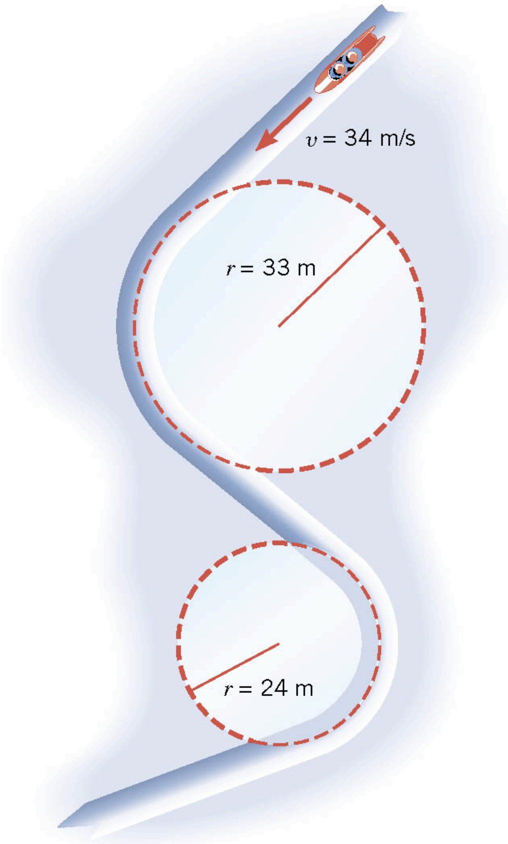
Example 5: Figure 1: An overhead view of a ball moving in a circular path in a horizontal plane. A centripetal force F_r directed toward the center of the circle keeps the ball moving in its circular path.

Figure 2 : Overhead view of a ball moving in a circular path in a horizontal plane. Then the string breaks. The ball leaves tangent to the circular path.

Figure 1: ball in a circular path	**Figure 2: The string breaks.**
	

Example 6: *A Bobsled;*

The bobsled track contains turns with radii of 33 m and 24 m with a speed of 34 m/s. The radius affects the centripetal acceleration at each turn. We can apply the formula: $a = v^2 / r$ for each turn.



Example 7: *A Trapeze Act*

A Trapeze Act In a circus, a man hangs upside down from a trapeze, legs bent over and arms downward, holding his partner. It is harder for the man to hold his partner when the team is swinging through the straight-down position than when they are hanging straight down and stationary.

When hanging straight down and stationary, the man has to support his partner's weight only. But, when the two are swinging, the partner is moving in a circular path and has a centripetal acceleration. Then, the man's arms must exert an additional pull or centripetal force to produce this acceleration.

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The edition was dedicated to the memory of Stella Kupferberg, Director of the Photo Department: "We miss you, Stella, and shall always remember that a well-chosen photograph should speak for itself, without the need for a lengthy explanation"

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