Motion along an Inclined Plane

by

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P3.2 Net Forces

Forces have magnitude and direction. The net force on an object is the sum of all the forces acting on the object. Objects change their speed and/or direction only when a net force is applied. If the net force on an object is zero, there is no change in motion (Newton's First Law).

P3.2d Calculate all the forces on an object on an inclined plane and describe the object's motion based on the forces using free-body diagrams.

P3.1d Identify the basic forces in everyday interactions.

Items:

1- Inclined Plane and Newton’s Second Law of Motion.

2- Objects at Rest and in Motion Along an Inclined Plane.

3- Objects in Motion Along an Inclined Plane.

4- Analysis of the Acting Forces along the Axis

5- Analysis of the Acceleration along the Axis.

Newton's Second Law of Motion

When a net external force acts on an object of mass **m**, the acceleration that results is directly proportional to the net force and inversely proportional to the mass.

Acceleration = Net Force / Mass

Net Force = Mass x Acceleration

The direction of the acceleration is the same as the direction of the net force.

The acceleration of an object increases when the net external force increases.

The acceleration decreases when the mass of the object increases.

F net: It is the sum of all the external forces acting on the object in Newton (N):

Newton = $(\mathrm{kg})\left(\frac{\mathrm{m}}{\mathrm{s}^2}\right) = \frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}^2}$

The direction of force and acceleration vectors can be taken into account by using x and y components:

Net Force Components along the x and y Axis
Net Force = Mass x Acceleration $\vec{F}_{net} = m \times \vec{a}$ is equivalent to: F_x net = m a_x (along the x axis) F_y net = m a_y (along the y axis)

Newton's Second Law of Motion Application on Inclined Plane

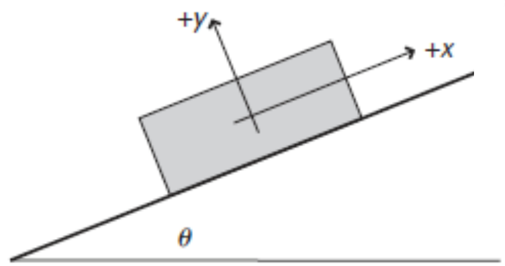
An example of an inclined plane is a hill going up with a slope.

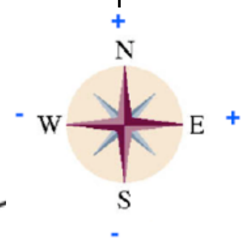
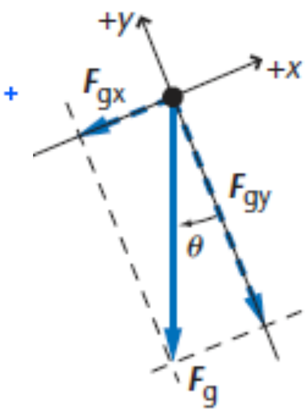
Object at Rest on the Hill	Object in Motion up the Hill
 A rock resting or not moving on a hill.	 The vehicle in the picture in going up the hill. The blue arrow is the gravitational force Fg, directed toward the center of the earth.

I.

Object at Rest (not Moving) on an Inclined Plane

Below you can see 3 representations (A, B and C) of rock resting on a hill or inclined plane:

<table>
<tr><th>A) A rock resting or not moving on a hill sloped at an angle θ</th><th>B) A physical representation of the rock resting on the hill.</th><th>C) Free body-diagram of the rock resting on the hill.</th></tr>
<tr><td></td><td>


</td><td>
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A) A rock is resting on a hill with an angle θ above the horizontal.

B) In the coordinate system, the positive x axis is pointing uphill. The y axis is usually perpendicular to the x axis, or perpendicular to the surface of the hill.

C) The free-body diagram shows the weight (w) of the rock represented by the gravitational force (F_g) pointing to the center of the earth.

A *free-body-diagram* is a diagram that represents the object and the forces that act on it.

F_g has two components:

1- one along the x axis (F_{gx}) and

2- one along the y axis (F_{gy}).

Analysis of Gravitational Force Components

(Object at Rest on an Inclined Plane)

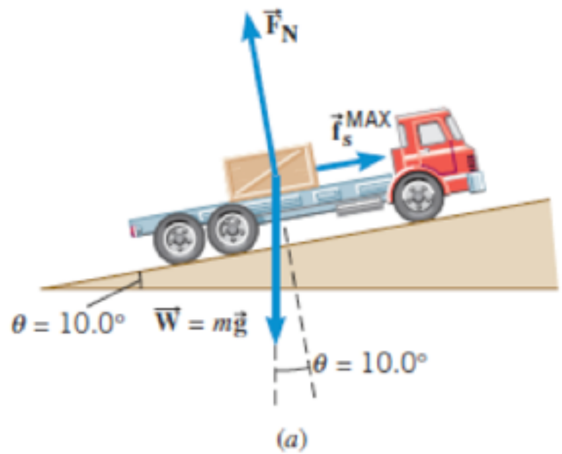
F_g has two components: one along the x axis (F_{gx}) and one along the y axis (F_{gy}). The angle (θ) equals the inclined angle of the hill.

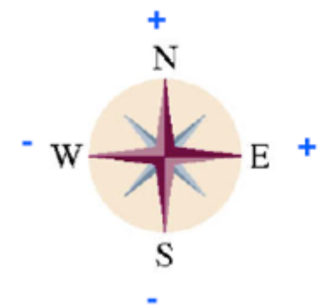
The mathematical values of both components is shown in the table below:

Gravitational Force (Fg) **and its Components along the x axis and y axis**
Component along the x axis: $F_{gx} = -F_g \sin\theta$
Component along the y axis: $F_{gy} = -F_g \cos\theta$

Object Moving on an Inclined Plane.

A flatbed truck carrying a crate up a hill with an angle θ (10^o) above the horizontal is motion along an inclined plane.





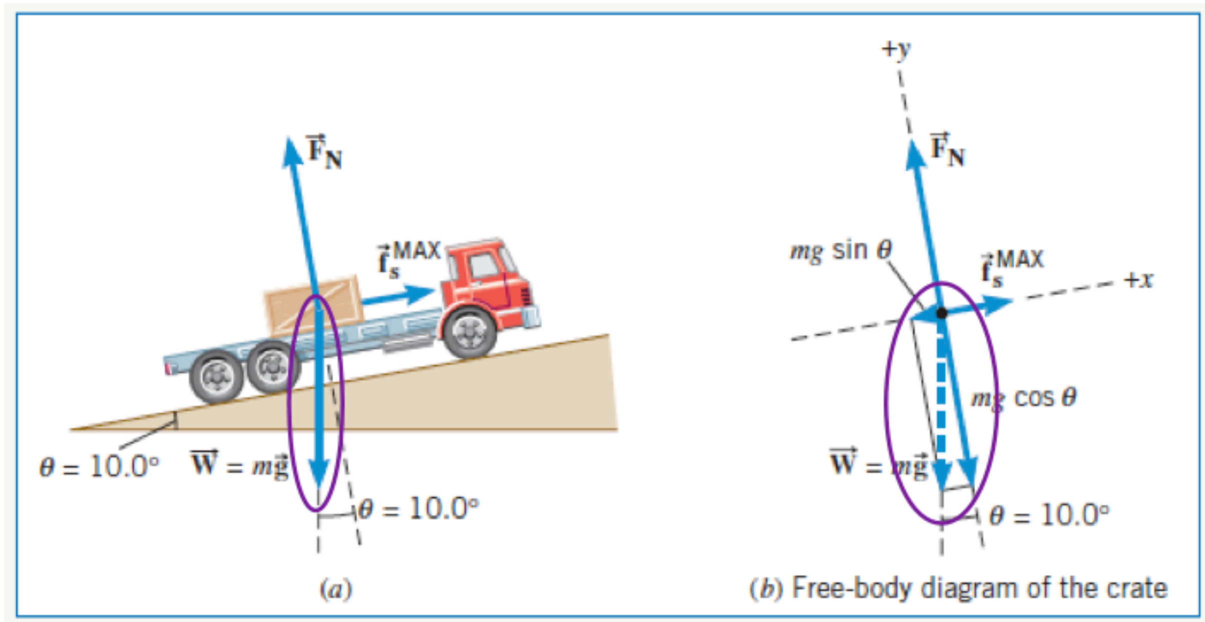
(a) There are three vector forces acting on the crate as shown by blue arrows.

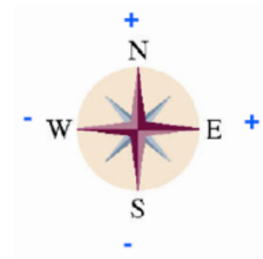
1) The normal or support force pointing north perpendicular to the hill ($+ F_N$).

2) The weight of the crate pointing down south toward the center of the Earth south, so it is negative, - W = *mg*.

3) There is also the frictional force f_s^{MAX} pointing toward the east, so it is positive $+ f_s^{MAX}$.

Normal (Support) Force and Weight Equilibrium on a hill





b) The free body diagram for the crate. Because it is an inclined motion, the weight W is divided into two components along the x and y axis.

As indicated by the purple ovals. The weight (W = mg) is divided into two components: one component along the x axis ($W_x = - m\,g\sin\theta$) and another component along the y axis ($W_y = - m\,g\cos\theta$).

Summary Table of the Forces acting on the Crate along the x axis and y axis:

(Object Moving along an Inclined Plane)

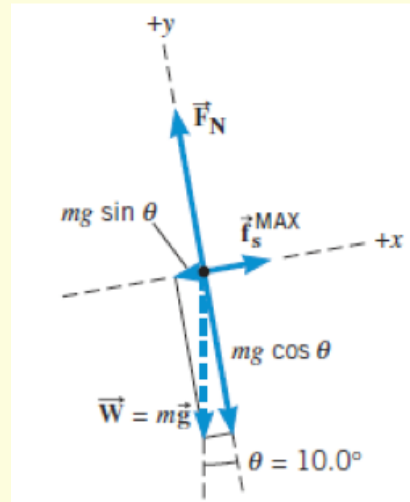
Along the x axis	Force	Direction
Force 1	Weight component: $W_x = - m g \sin \theta$	backward and negative
Force 2	Static frictional force f_s^{MAX}	forward and positive

Along the y axis	Force	Direction
Force 1	Weight component: $W_y = - m g \cos \theta$	downward and negative and perpendicular to the hill
Force 2	Normal or Support force F_N	upward and positive and perpendicular to the hill

Moving Object: Forces along the x axis and their sum (F_x net)

Forces along the x axis and their sum (F_x net)

W_x component along the x axis (backward, negative):



$$W_x = - m\, g \sin\theta$$

The friction force f_s^{MAX} (forward, positive):

$$f_s^{MAX} = \mu_s F_N = \mu_s\, m\, g \cos\theta$$

The sum of the forces F_x net:

$$F_x \text{ net} = W_x + f_s^{MAX} = m\, g\, (\mu_s \cos\theta - \sin\theta)$$

Acceleration a_x:

$$a_x = F_x \text{ net} / m = m\, g\, (\mu_s \cos\theta - \sin\theta)/m$$

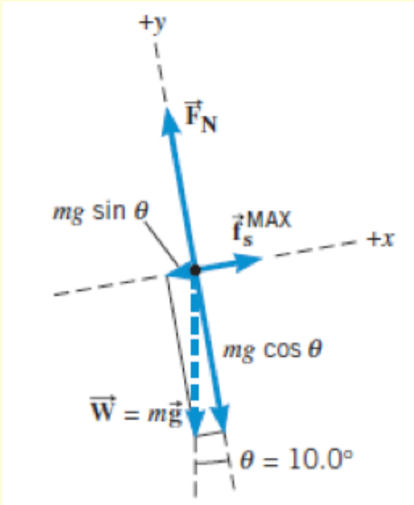
$$= g\, (\mu_s \cos\theta - \sin\theta)\ \text{m/s}^2$$

Moving Object: Forces along the Y axis and their sum (F_y net)

Forces along the Y axis and their sum (F_y net)

W_y component along the y axis (down, negative):

$$W_y = - m\, g \cos\theta$$



The normal or support force F_N (up, positive)

$$F_N = -W_y = m\, g \cos\theta$$

The sum of the forces F_y net:

$$F_y \text{ net} = W_y + F_N = 0 \text{ Newton}$$

Acceleration a_y:

$$a_y = F_y \text{ net}/ m = 0/m = 0 \text{ m/s}^2$$

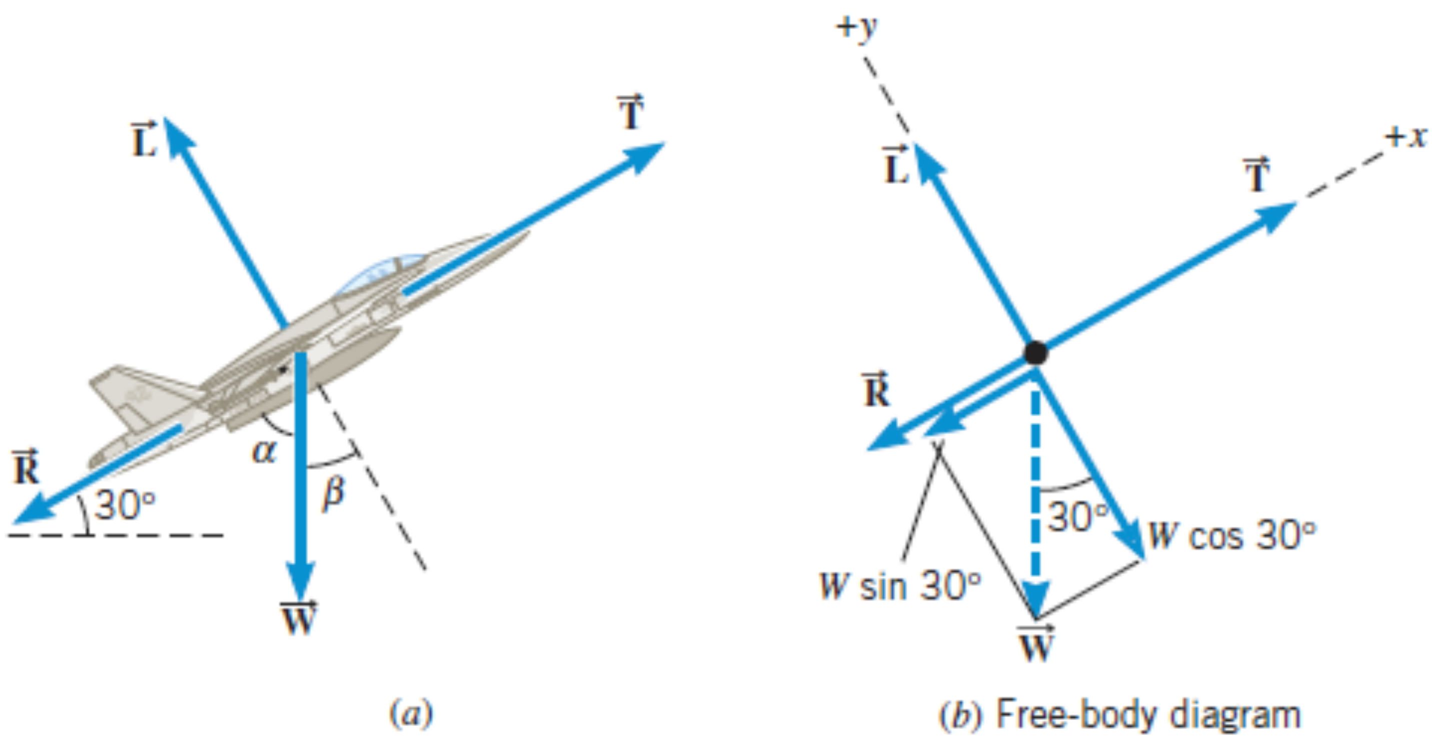
Summary Table for Acceleration:

Acceleration along an Inclined Plane Summary **Moving Object**	
Acceleration a_y:	$a_y = 0\ m/s^2$
Acceleration a_x: Notice that the mass is not in the formula.	$a_x = g\ (\ \mu_s \cos\theta - \sin\theta\ )$

Example 1: *A Jet Plane.*

a) A plane moves with constant velocity at an angle of 30.0 º (θ = 30.0 º) above the horizontal due to the action of four forces, the weight W , the lift L , the engine thrust T, and the air resistance R.

b) The free-body diagram of the plane. The black dot represent the plane.



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The edition was dedicated to the memory of Stella Kupferberg, Director of the Photo Department: "We miss you, Stella, and shall always remember that a well-chosen photograph should speak for itself, without the need for a lengthy explanation"

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