Projectile Launched Horizontally

by

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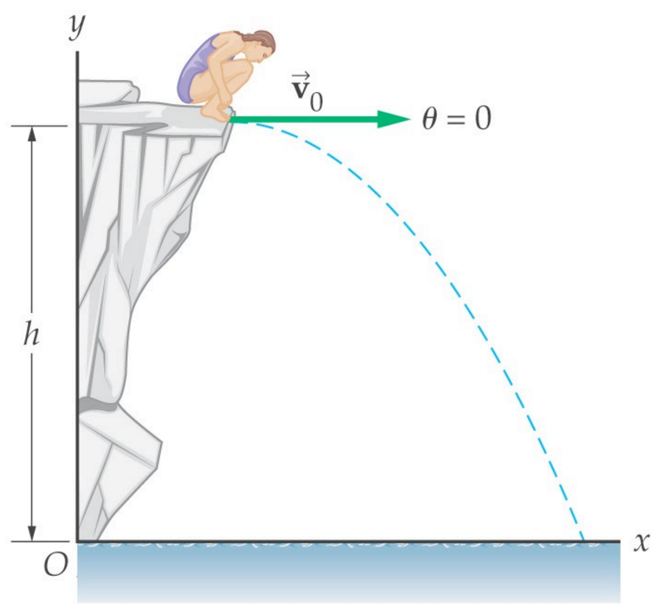
P3.3c Explain the recoil of a projectile launcher in terms of forces and masses.

P3.4e Solve problems involving force, mass, and acceleration in two-dimensional projectile motion restricted to an initial horizontal velocity with no initial vertical velocity (e.g., a ball rolling off a table).

Projectile Launched Horizontally

Projectile launched at zero angle ($\theta = 0$) is a free fall with an initial horizontal velocity (**V**$_o$).

This is an example of projectile launched horizontally with an angle $\theta = 0$ comparing to the horizontal.



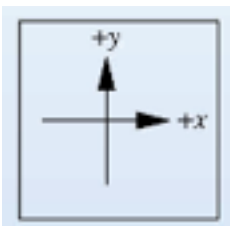
Characteristics of Projectile Motion

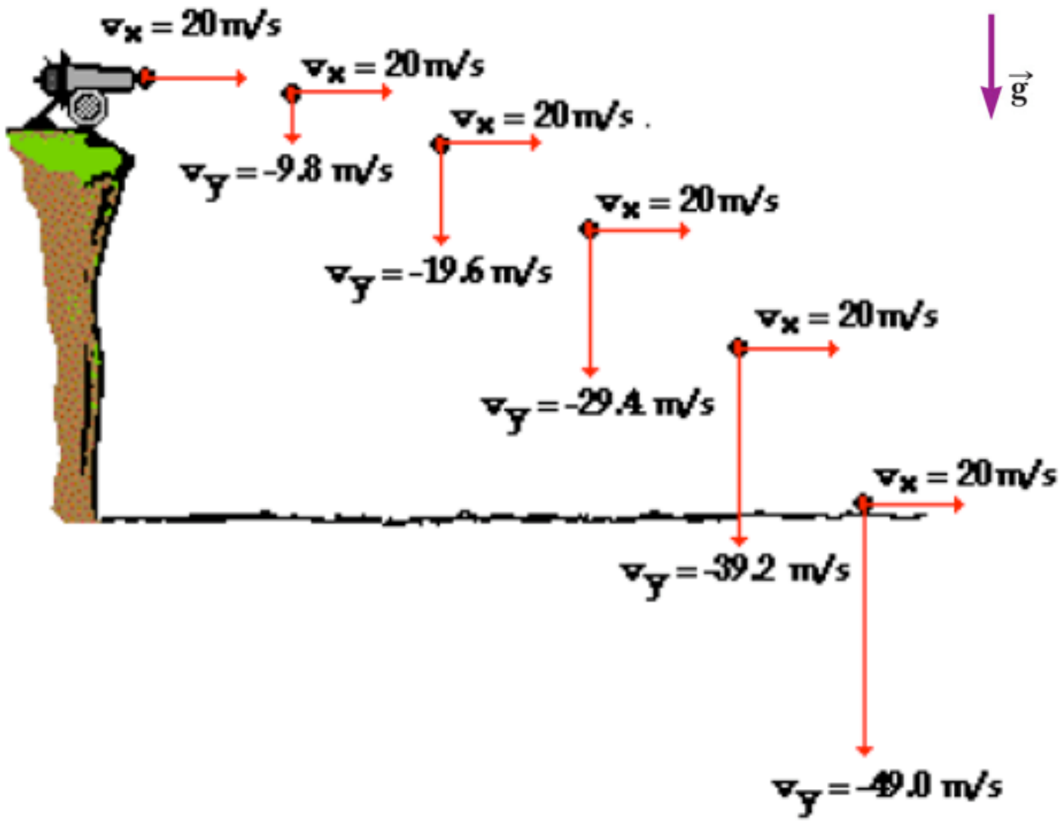
- Projectile motion is the motion of objects moving in two dimensions horizontal (v_x) and vertical (v_y).
- In Projectile motion:
  - the horizontal motion and the vertical motion are independent of each other. Neither motion affects the other.
  - the initial vertical velocity is zero ($v_{oy} = 0$), but increases regularly because of the acceleration due to gravity (g). So, v_y increases along the path.
  - the initial horizontal velocity (v_{ox}) stays the same along the path ($v_{ox} = v_x =$ constant).

Example 1: Below is an example of projectile launched horizontally.

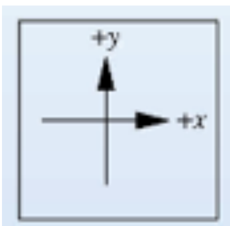
a) Notice that the initial horizontal velocity (v_{ox}) stays the same along the path

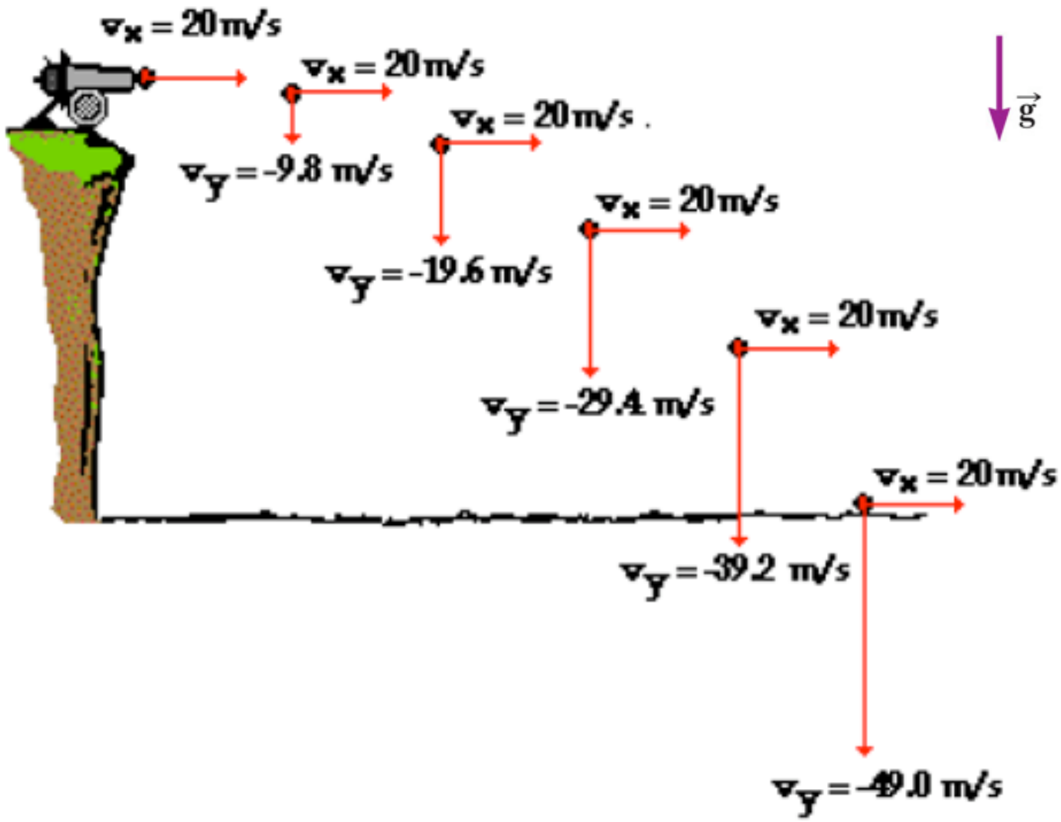
($v_{ox} = v_x$ = constant = 20 m/s).





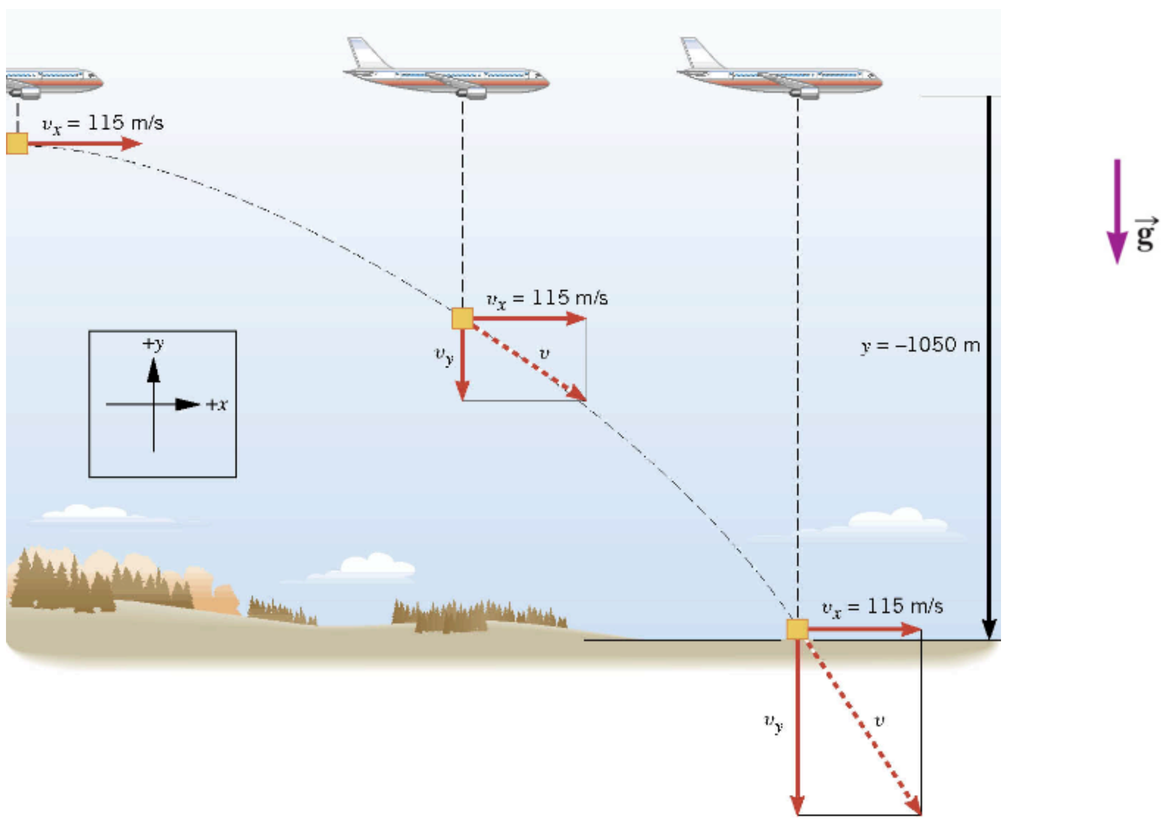
b) Notice that at launch there is no initial vertical velocity (v_{oy} = 0). Later; the vertical velocity v_y increases along the path from -9.8 m/s to -19.8 m/s to -29.4 m/s to -39.2 m/s to -49.0 m/s when it reaches the ground.





Example 2: *A Falling Care Package:*

The figure below shows airplane moving horizontally with a constant velocity of +115 m/s at an altitude of 1050m. The direction to the right and upward have been chosen as the positive directions. The plane releases a "care package" that falls to the ground along a curved trajectory.



Notice that the care package remains directly under the plane all the time, as the dashed line shows for the following reasons:

a) Both the plane and care package travel at a constant velocity $v_{ox} = v_x =$ 115 m/s.

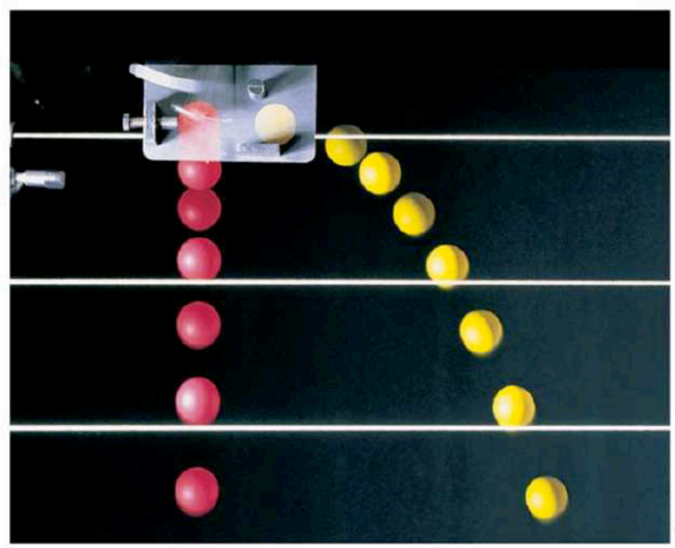
b) the initial vertical velocity is zero (v_{oy} = 0), but increases regularly because of the acceleration due to gravity (g). So, v_y increases along the path.

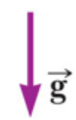
c) the horizontal motion and the vertical motion are independent of each other. Neither motion affects the other.

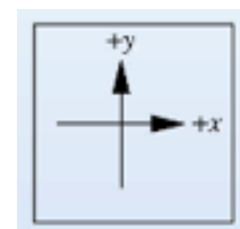
Vertical motion of projectile and free fall

The vertical motion of projectile launched horizontally is identical to that of a dropped object.

Verification: A strobe-light photo of two balls released simultaneously from a mechanism that allows one ball to drop freely (red) while the other is projected horizontally (yellow).

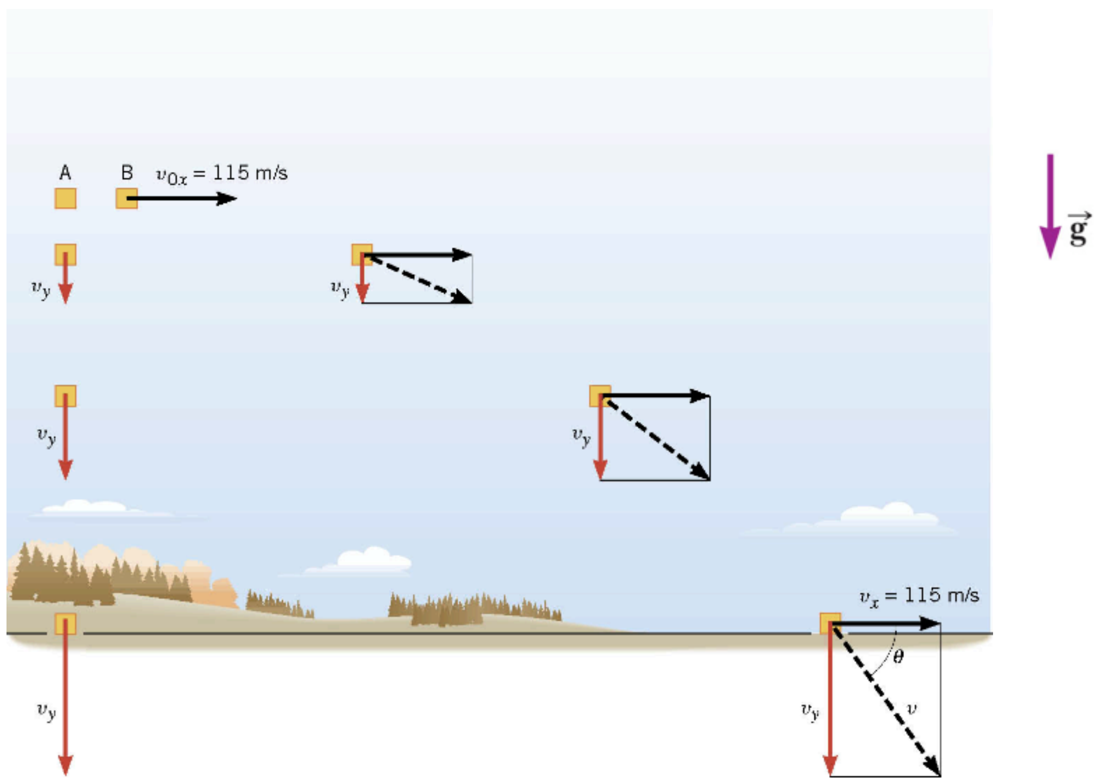






Notice that at equal times, both balls fall the same vertical distance.
Both hit the ground at the same time.

Example 3: The figure below shows two packages A and B. Package B is given an initial velocity component Vx = 115 m/s in the horizontal direction. The package follows a path as shown in the figure. Package A, on the other hand, is dropped from a stationary balloon and falls straight down toward the ground (Vx = 0 m/s). Both packages A and B hit the ground at the same time.



Kinematic Equations for Projectile Motion

The basic kinematics equations are used with the components of motion.

X axis:

Variable along the x axis: x, v_x, v_{ox}, a_x , t

Horizontal Component of Motion:

1. $a_x = 0$ (acceleration along the x axis)
2. $v_x = v_{ox}$ = constant

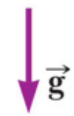
Kinematic equations: Projectile launched horizontally at an angle $\theta = 0$	
X components: Velocity component along the x axis (v_{ox})	$v_{ox} = v_o$ = constant
Range R or Landing site x	$X = R = v_{ox}\, t$
Notice that the horizontal velocity component (v_{ox}) does not change because it is not a function of time.	

y axis:

Variable along the y axis: y, v_y, v_{oy}, a_y , t

Horizontal Component of Motion:

1. $a_y = g$ (- 9.8 m/s^2) = acceleration due to gravity directed down toward the center of the Earth.



2. v_{oy} is not constant

Kinematic equations: Projectile launched horizontally at an angle $\theta = 0$	
Y components;	
Velocity components along the y axis at different times (v_y).	$v_y = g\ t$
Maximum height launched y or H	$H = y = \frac{1}{2} g\ (t)^2$
t; time	

Time and total velocity

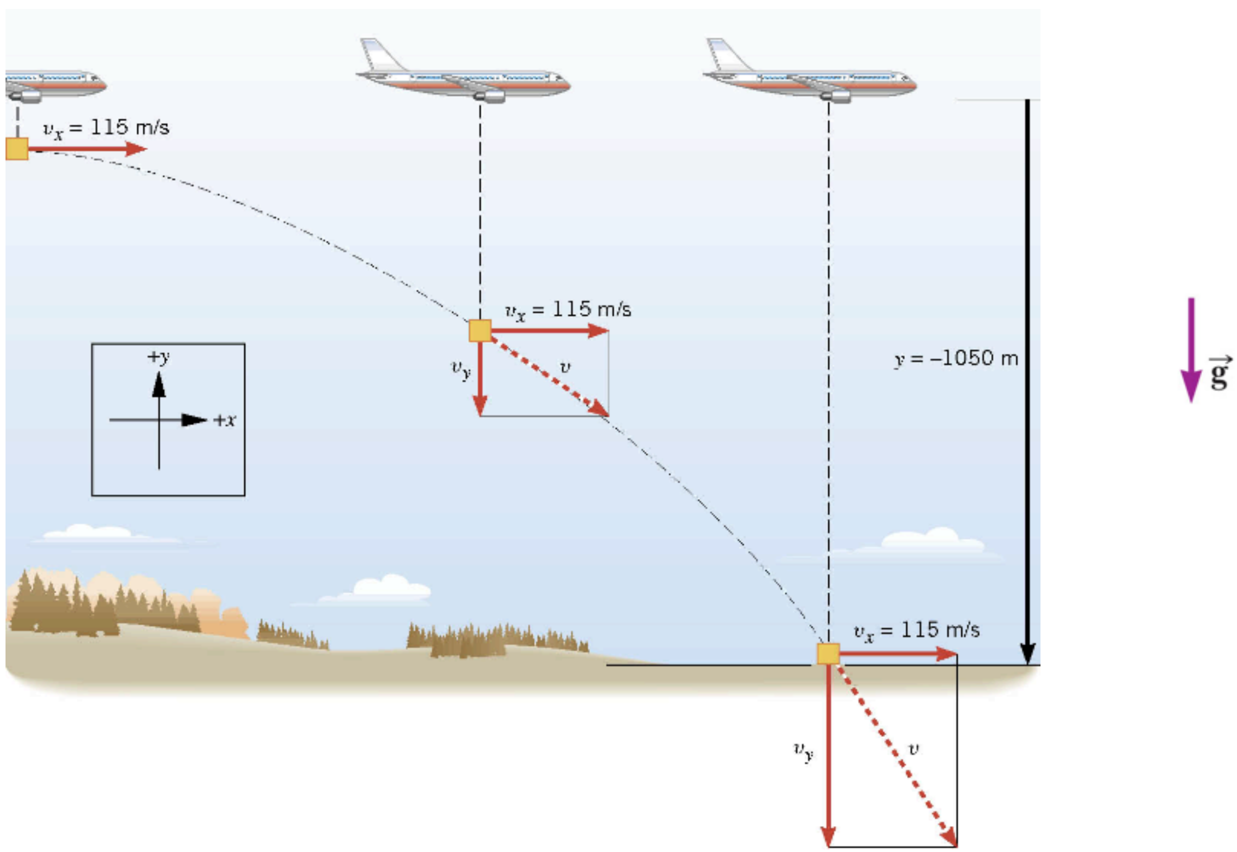
Kinematic equations: Projectile launched horizontally at an angle $\theta = 0$	
Time to reach the ground or "hang time"	$t^2 = 2y / g$
The total velocity at a specific instant	$(v)^2 = (v_o)^2 + (v_y)^2$

The angle the projectile reaches the ground

Kinematic equations: Projectile launched horizontally at an angle $\theta = 0$	
The angle (θ) the projectile reaches the ground tangent = tan.	$\tan\theta = v_y / v_x$ or $\theta = \tan^{-1}(v_y / v_x)$ $\text{Cos}\,\theta = v_x / v$ or $\theta = \text{Cos}^{-1}(v_x / v)$

Example 4: *A Falling Care Package:*

The figure below shows airplane moving horizontally with a constant velocity of +115 m/s at an altitude of 1050m. The direction to the right and upward have been chosen as the positive directions. The plane releases a “care package” that falls to the ground along a curved trajectory.



Ignoring air resistance, determine:

a- the time required for the care package to hit the ground (t)

b- the landing site of the care package, or range (R)

c- the vertical velocity of the care package when it hits the ground (Vy)

d- the final velocity of the care package (V).

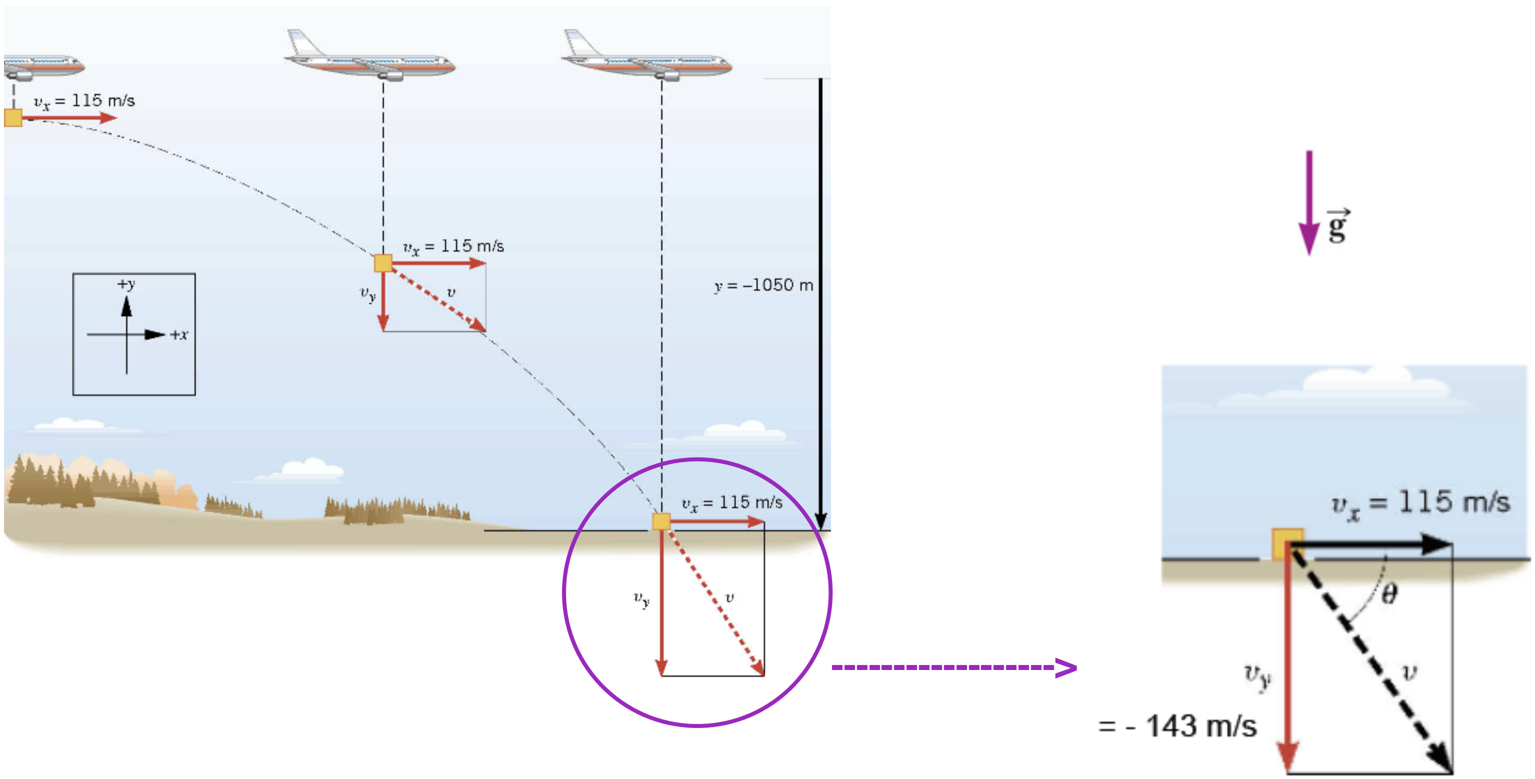
We know that acceleration along the y axis is a_y = g = - 9.8 m/s²

Data Table						
v_o	a_y	y	t	x	v_y	v
115 m/s	g = - 9.8 m/s^2	-1050 m	?	R?	?	?

Equations for Projectile Launched Horizontally ($\theta = 0$)

v_{ox} : Velocity component along the x axis, always the same.	$v_{ox} = v_o = +115$ m/s
t: time to reach the ground.	$t^2 = 2y/g = 2(-1050)/-9.8 \text{ m/s}^2 = 214.28 \text{ s}^2$ $t = 14.6$ s
X: Landing site	$x = v_{ox}t = (115 \text{ m/s})(14.6 \text{ s}) = 1679$ m
v_y :velocity components along the y axis at different times.	$v_y = g\,t = (-9.8 \text{ m/s}^2)(14.6 \text{ s}) = -143$ m/s negative because it is directed down.
Total velocity the instant before reaching the ground, or at any instant	$(v)^2 = (v_o)^2 + (v_y)^2$ $= (115 \text{ m/s})^2 + (-143 \text{ m/s})^2$ $= 33700 \text{ (m/s)}^2$ $v = 184$ m/s

We can calculate the angle θ when the care package hits the ground.



$$\theta = \tan^{-1}(v_y / v_x) = \tan^{-1}(143/115) = 51.2^o$$

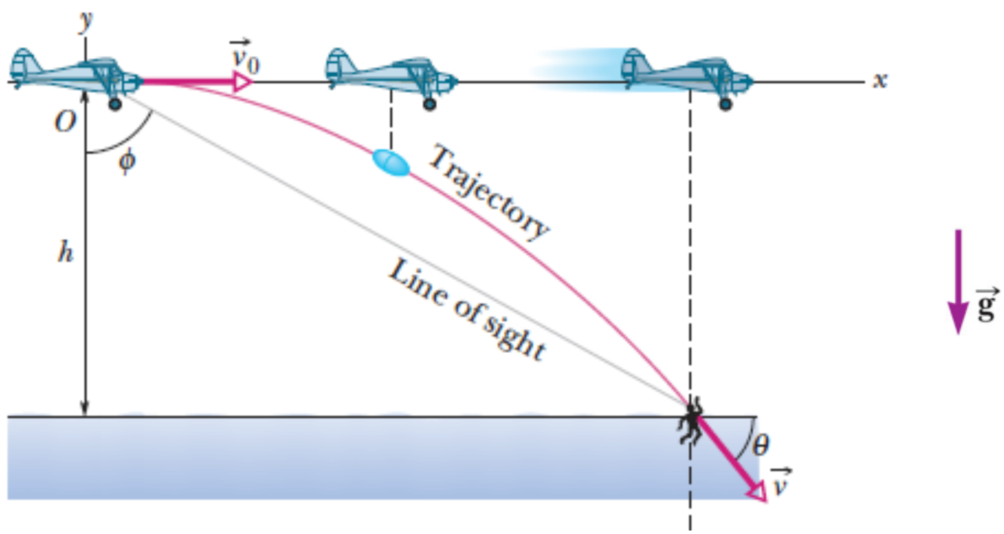
or

$$\theta = \cos^{-1}(v_x / v) = \cos^{-1}(115/184) = 51.3^o$$

Example 5:

Example 6: *A Rescue Plane*

A rescue plane flies at constant horizontal velocity of 55.0 m/s and constant height of 500 m toward a point directly over a victim, where a rescue capsule is to land. The plane drops a rescue capsule. While falling the capsule remains under the plane.



Data Table						
v_o	a_y	y	t	x	v_y	v
55 m/s	g = - 9.8 m/s^2	-550 m	?	R?	?	?

References:

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Physics 1200 Lecture Slides: Dr. Thomas Humanic, Professor of Physics, Ohio State University, *2013-2014 and Current*. *www.physics.ohio-state.edu/~humanic/*

2) Cutnell, J. D. & Johnson, K. W. (1998). *Cutnell & Johnson Physics, Fourth Edition*. New York: John Wiley & Sons, Inc.

The edition was dedicated to the memory of Stella Kupferberg, Director of the Photo Department:"We miss you, Stella, and shall always remember that a well-chosen photograph should speak for itself, without the need for a lengthy explanation"

3) Martindale, D. G. & Heath, R. W. & Konrad, W. W. & Macnaughton, R. R. & Carle, M. A. (1992). *Heath Physics.* Lexington: D.C. Heath and Company

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7) Nada H. Saab (Saab-Ismail), (2009- 2014) Wayne RESA, Bilingual Department.