Projectile Launched at an Angle

by

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P2.2g Apply the independence of the vertical and horizontal initial velocities to solve projectile motion problems.

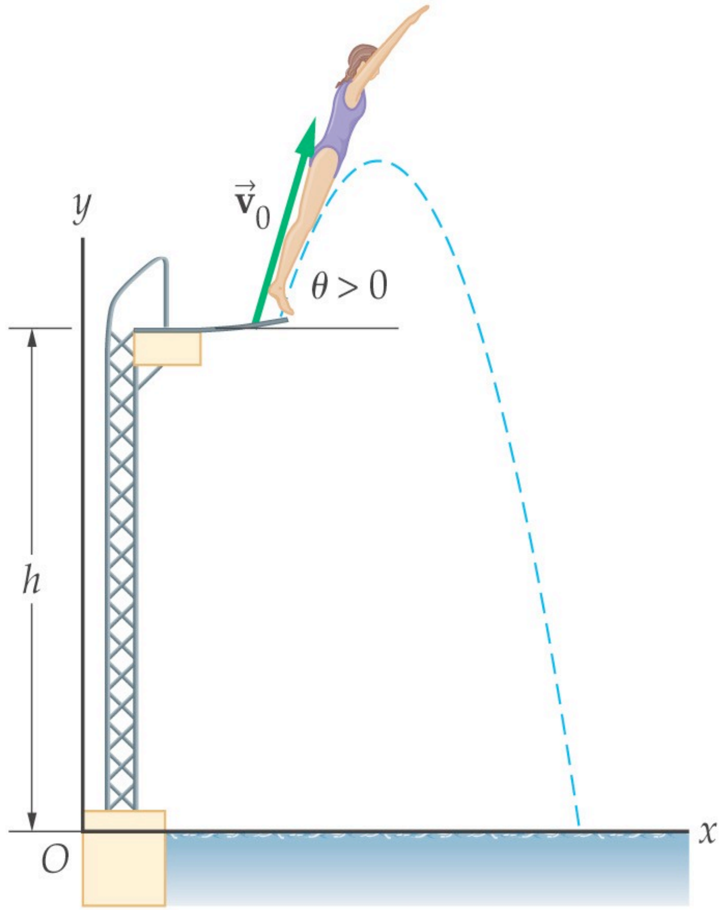
P3.4f Calculate the changes in velocity of a thrown or hit object during and after the time it is acted on by the force.

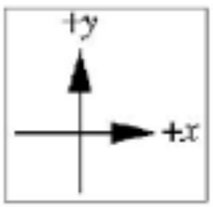
Items:

1. Variable along the x axis: x, v_x, v_{ox}, a_x, t
2. Variable along the y axis: y, v_y, v_{oy}, a_y, t
3. Maximum Height Reached.
4. Hang Time.
5. Range.

Projectile Launched at an Angle

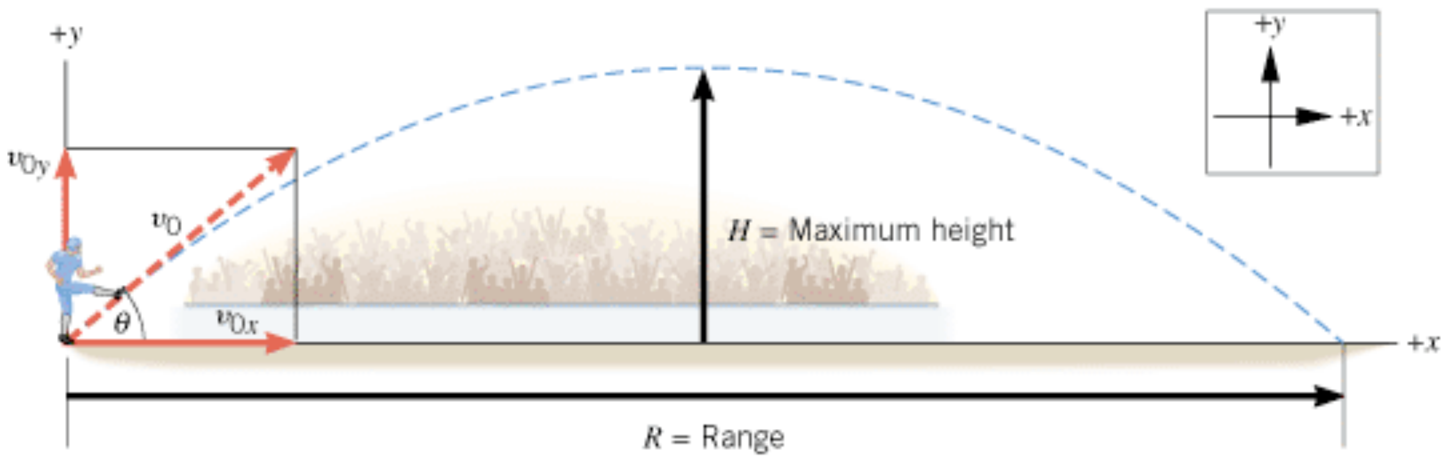
Diving with an initial velocity ($\mathbf{v}_o$) at an angle $\theta > 0$ above the horizontal is an example of projectile launched at an angle.





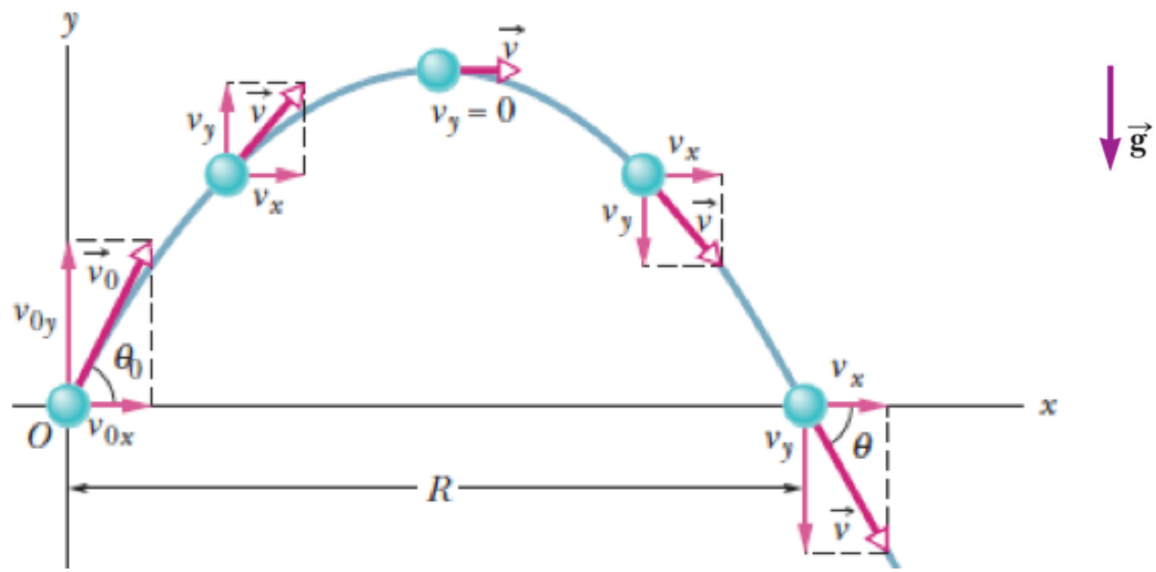
Projectile Launched at an Angle

Kicking a football with an initial velocity (v_o) at an angle (θ) above the horizontal is another example of projectile launched at an angle.



Path and velocity components

The diagram below shows the entire path of the football.



We can see the velocity (v) at certain points along the trajectory. Notice the corresponding horizontal (v_x) components of the velocity along the x axis and vertical (v_y) components of the velocity along the y axis. R is the range.

Kinematic Equations for Projectile Motion

The basic kinematics equations are used with the components of motion.

X axis:

Variable along the x axis: x, v_x, v_{ox}, a_x, t

Horizontal Component of Motion:

1. $a_x = 0$ (acceleration along the x axis)
2. $v_x = v_{ox}$ = constant

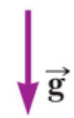
Kinematic equations: Projectile launched at an angle θ	
X components: Velocity component along the x axis (v_{ox})	$v_{ox} = v_o \cos\theta$
Range R or Landing site x	$X = R = v_{ox}\, t$
Notice that the horizontal velocity component (v_{ox}) does not change because it is not a function of time.	

y axis:

Variable along the y axis: y, v_y, v_{oy}, a_y , t

Horizontal Component of Motion:

1. $a_y = g$ (9.8 m/s^2) = acceleration due to gravity directed down toward the center of the Earth.



2. v_{oy} is not constant

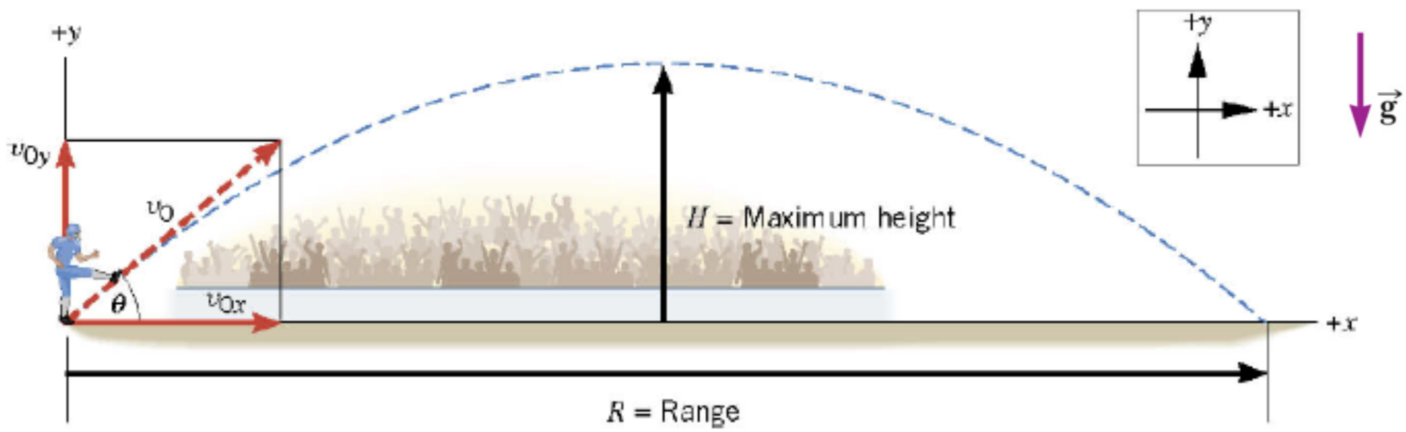
Kinematic equations: Projectile launched at an angle θ	
Y components;	$v_{oy} = v_o \sin\theta$
Velocity components along the y axis at different times (v_y).	$v_y = v_{oy} + g\ t$
Maximum height reached y or H	$H = y = v_{oy}\, t\ + \frac{1}{2}\, g\ (t)^2$
t; time	$(v_y)^2 = (v_{oy})^2 + 2\, g\ y$

Time and total velocity

Kinematic equations: Projectile launched at an angle θ	
Time to reach the ground or “hang time”	$t = -2\, v_{oy} / g$
Time at maximum height	$t = -\, v_{oy} / g$
The total velocity at a specific instant	$(v)^2 = (v_o)^2 + (v_y)^2$

Example 1: A Player Kicking a Football at an Angle (θ= 30.0°).

A player kicks a football from ground level with a velocity (v_o) of magnitude 27.0 m/s (v_o = 27.0 m/s) at an angle (θ) of 30.0° above the horizontal (θ= 30.0°).



Calculate:

a. the horizontal (v_{ox}) and vertical (v_{oy}) components of the initial velocity (v_o).

b. its " hang time" (t_1) or the time the ball is in the air, or the time to reach the ground.

c. the distance the ball travels before it hits the ground or range (R).

d. the time at maximum height (t_2) and its maximum height (H).

Data Table								
v_o	$a_y = g$	θ	t_1	y	t_2	x	v_{ox}	v_{oy}
+27 m/s	- 9.8 m/s ^{2}	30.0°	?	H?	?	R?	?	?

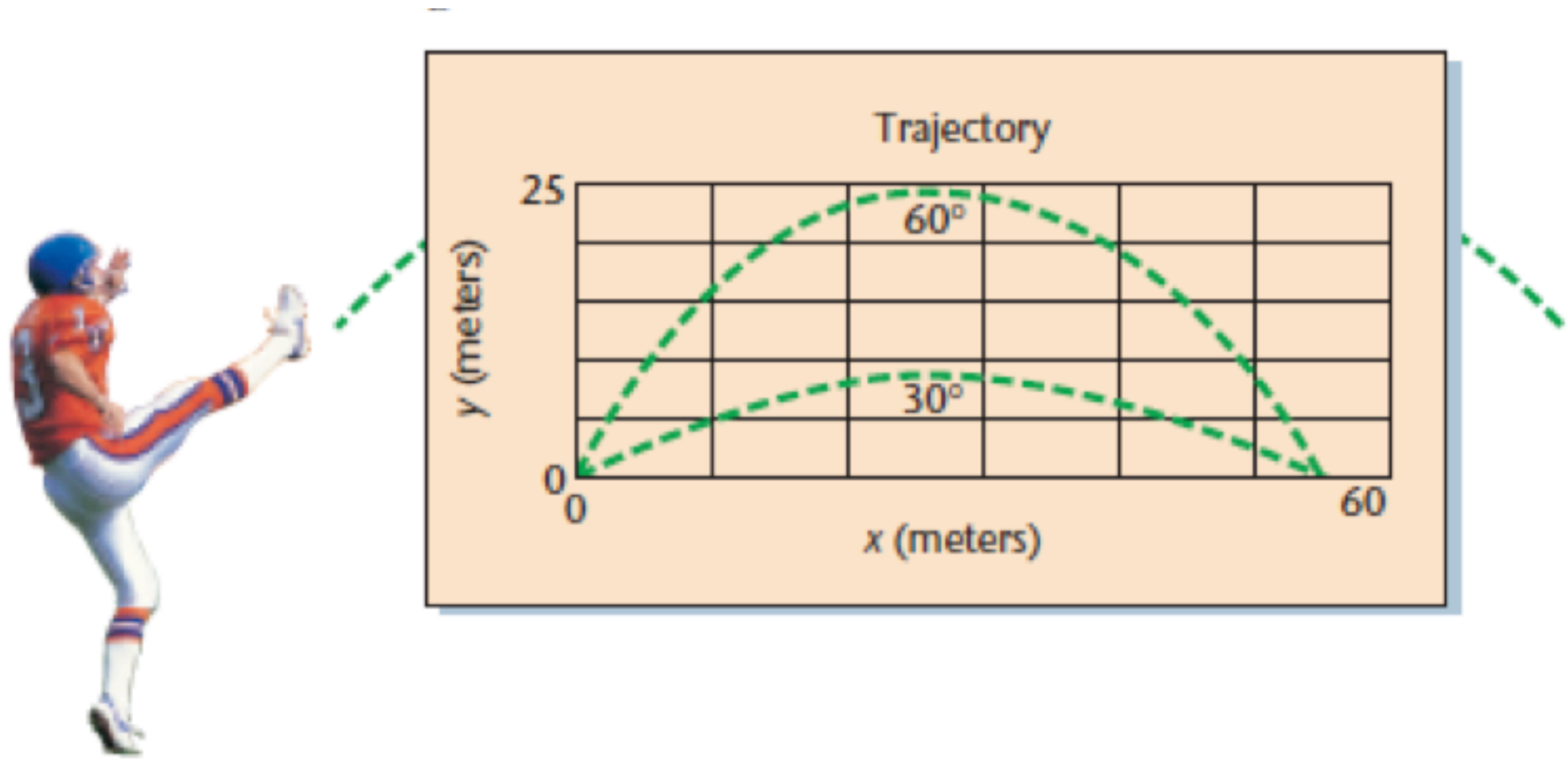
Calculation:

Projectile launched at an angle θ	
Velocity component along the x axis v_{ox}	$v_{ox} = v_o \cos\theta =$ (27.0) cos 30.0° = 23.4 m/s
Velocity component along the y axis v_{oy}	$v_{oy} = v_o \sin\theta =$ (27.0) sin 30.0° = 13.5 m/s
Hang Time t_1	$t_1 = -2\, v_{oy} / g =$ - 2 (13.5) / (-9.8) = 2.76 s
Distance the ball travels, range, X or R	$x = v_{ox}\, t_1 =$ (23.4)(2.76) = 64.6 m
Time at maximum height $t_2 = t_1 / 2$	$t_2 = -\, v_{oy} / g =$ - (13.5) /(-9.8) = 1.38 s or $t_2 = t_1 / 2 =$ 2.76/ 2 = 1.38 s
maximum height y or H	$y = v_{oy}\, t_2 + \frac{1}{2} g\, (t_2)^2$ = (13.5)(1.38) + (- 9.8)(1.38)^2 / 2 = 18.6 - 9.3 = 9.3 m.

Notice that time at maximum height occurs at half the "hang time" or 2.76 s/ 2 = 1.38 s.

Example 2: The Same Player Kicking a Football at Different Angle (θ= 60.0°)

The same kicker now kicks the football from ground level with a velocity of magnitude 27.0 m/s (v_o = 27.0 m/s) but at an angle of 60.0° above the horizontal (θ= 60.0°).



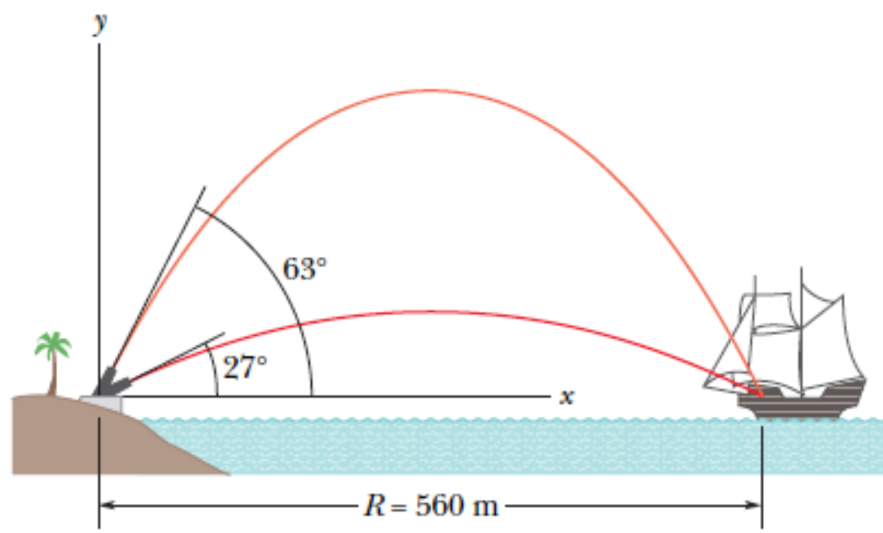
The horizontal (v_{ox}) and vertical (v_{oy}) components of the initial velocity (v_o), the " hang time" (t), the range (R), the time at maximum height and its maximum height (H) can be calculated as for Example 1. The values are summarized in the table below.

Data Table							
θ	v_o	$a_y = g$	v_{ox}	v_{oy}	t_1	$y = H$	$x = R$
30.0°	27 m/s	9.8 m/s ^{2}	23.4 m/s	13.5 m/s	2.76 s	9.3 m	64.6 m
60.0°	27 m/s	9.8 m/s ^{2}	13.5 m/s	23.4 m/s	4.78 s	27.9 m	64.5 m

When the launch angle doubled, the time to reach maximum height doubled, but the range remained the same.

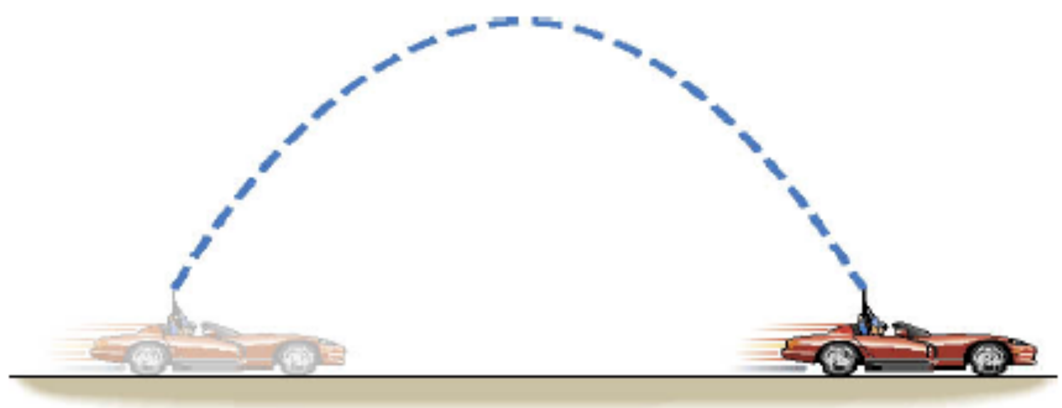
Example 2: *A Pirate Ship*

The figure shows a pirate ship 560 m from a fort defending a harbor entrance. A defense cannon, located at sea level, fires a ball at initial speed Vo = 82 m/s.



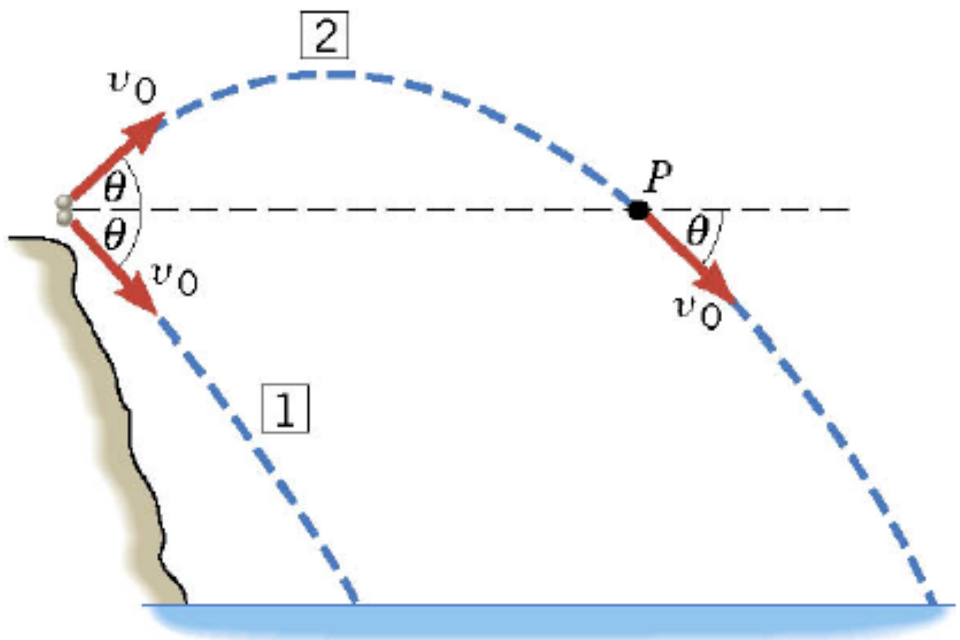
Example 3: *Bullet Into the Air.*

Suppose you are driving a convertible with the top-down. The car is moving to the right at constant velocity. Air resistance is ignored. You point a rifle straight up into the air and fire it. Below is the trajectory of the bullet.



Example 4: *Two Ways to throw a stone,*

From the top of a cliff overlooking a lake, a person throws two stones. The stones have identical initial speeds, but stone 1 is thrown downward at some angle θ below the horizontal, while stone 2 is thrown upward at the same angle θ above the horizontal. Neglect air resistance. Compare the speed of the stones when they hit the water surface.



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2) Cutnell, J. D. & Johnson, K. W. (1998). *Cutnell & Johnson Physics, Fourth Edition*. New York: John Wiley & Sons, Inc.

The edition was dedicated to the memory of Stella Kupferberg, Director of the Photo Department: "We miss you, Stella, and shall always remember that a well-chosen photograph should speak for itself, without the need for a lengthy explanation"

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