Mirror Equations

by

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P4.8 Wave Behavior — Reflection and Refraction

The laws of reflection and refraction describe the relationships between incident and reflected/refracted waves.

P4.8A Draw ray diagrams to indicate how light reflects off objects or refracts into transparent media.

P4.8B Predict the path of reflected light from flat, curved, or rough surfaces (e.g., flat and curved mirrors, painted walls, paper).

P4.9B Explain how various materials reflect, absorb, or transmit light in different ways.

Items;

1- Mirror Equation

2- Magnification Equation

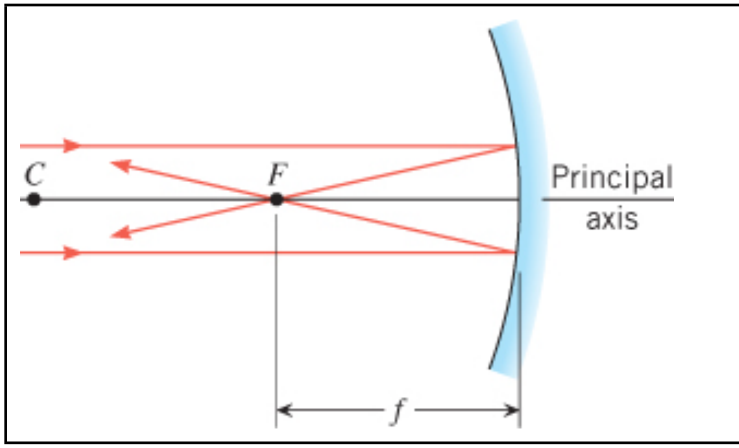
3- Summary of Sign Conventions for Spherical Mirrors

FOCAL LENGHT (convex and concave mirrors)

The focal point F of a concave mirror is halfway between the center of curvature C and the mirror.

The focal length f is the distance between the focal point and the mirror.

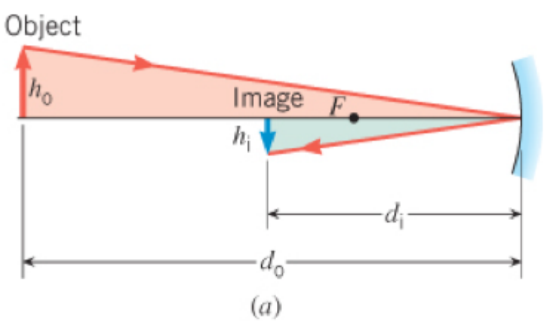
R is the radius of curvature of the mirror.



$$f = \frac{1}{2} R$$

MIRROR EQUATION (convex and concave)

Mirror equation is very useful, because it relates both image and object distances and the focal length.

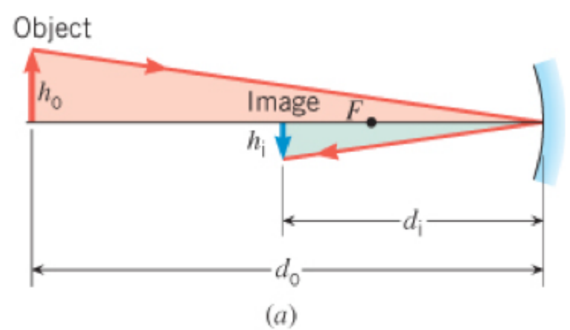
	Mirror Equation	
	$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$	f = focal length d_o = object distance d_i = image distance

Using mirror equation, the following relationships can be derived for image position, object position, and focal length:

$$d_i = \frac{fd_o}{d_o - f} \qquad d_o = \frac{fd_i}{d_i - f} \qquad f = \frac{d_i d_o}{d_o + d_i}$$

MAGNIFICATION EQUATION (convex and concave)

The magnification of an object by a spherical mirror, defined as the image height (h_i) divided by the object height (h_o) , is equal to the negative of the image position (d_i), divided by the object position (d_o).

	Magnification Equation
	$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$

If the image height is less than the object height $(h_i < h_o)$, the magnitude of m is less that one (**m < 1**).

If the image height is larger than the object height $(h_i > h_o)$, the magnitude is of m is greater than one (**m > 1**).

Summary of Sign Conventions for Spherical Mirrors

f is + for a concave mirror.

f is – for a convex mirror.

d_o is + if the object is in front of the mirror.

d_o is – if the object is behind the mirror.

d_i is + if the image is in front of the mirror (real image).

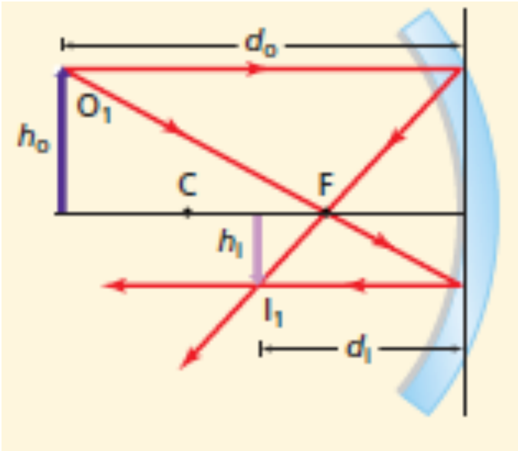
d_i is – if the image is behind the mirror (virtual image).

m is + for an image upright with respect to the object.

m is - for an image inverted with respect to the object.

Example 1: *Real Image Formation by a Concave Mirror;*

A concave mirror has a radius (R) of 20.0 cm. A 2.0-cm-tall object (h_o) is 30.0 cm from the mirror (d_o). What is the image position (d_i), the image height (h_i) and the magnification (m)?



From the tope of the object O_1 (**dark purple up arrow**) two rays are emitted:

A ray is drawn parallel to the principal axis is reflected through F.

A ray passes through F and is reflected parallel to the principal axis.

The two rays intersect in front of the mirror at point , top of the image I_1 (**light purple down arrow**).

Data Table						
d_o	h_o	R	$f = R/2$	d_i	h_i	m
30.0 cm	2.0 cm	20.0 cm	10.0 cm	?	?	?

The Image Position or Distance of the Image (d_i):

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$

$1/d_i$ = 1/10.0 - 1/ 30.0

= 0.1 - 0.033

= 0.067

So;

d_i = 1/ 0.067

= 15.0 cm

d_i is a positive number, so the image is real, in front of the mirror.

The Height of the Image (h_i):

$$h_i = \frac{-d_i h_o}{d_o}$$

h_i = - (15.0 x 2.0) / 30.0 = - 1.0 cm

The image height is negative, so it is inverted and smaller than the object.

The Magnification:

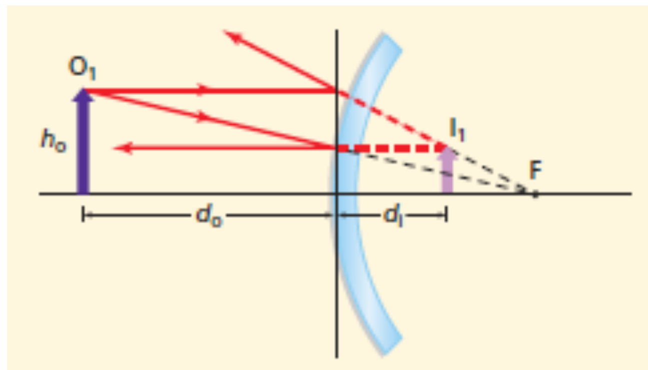
There are two ways to calculate the magnification:

1) The magnification m = h_i / h_o = -1.0 / 2.0 = - 0.5

2) The magnification m = - d_i / d_o = - 15.0 / 30/0 = - 0.5

Example 2: Image Formation by a Convex Security Mirror;

A convex security mirror in a warehouse has a -0.50-m focal length (f). A 2.0-m-tall (h_o) forklift is 5.0 m (d_o) from the mirror. What is the image position (d_i), the image height (h_i) and the magnification (m)?



The top point on the object O_1 (**dark purple up arrow**) emits two rays:

A ray is initially parallel to the principal axis and reflects from the mirror. The reflected ray appears to originate from the focal point (F), as shown when tracing its path with a dotted line behind the mirror.

A ray heads towards the focal point (F), emerging parallel to the principal axis after reflection. Its path is traced with a dotted line behind the mirror.

Both rays intersect behind the mirror to form the top point of a virtual image I_1 (**light purple up arrow**).

Data Table					
d_o	h_o	f	d_i	h_i	m
5.0 cm	2.0 cm	-0.5 m	?	?	?

The Image Position or Distance of the Image (d_i):

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$

$1/d_i$ = 1/ - 0.5 - 1/ 5.0

= - 2 - 0.2

= 2.2

So;

d_i = 1 / 2.2

= - 0.45 m

d_i is a negative number, so the image is virtual, behind the mirror.

The Height of the Image (h_i):

$$h_i = \frac{-d_i h_o}{d_o}$$

h_i = (- 0.45 x 2.0)/ - 5.0 = 0.18 m

The image height is positive, so it is upright and smaller than the object.

The Magnification:

There are two ways to calculate the magnification:

1) The magnification m = h_i / h_o = 0.18 / 2.0 = 0.09

2) The magnification m = - d_i / d_o = - (- 0.45) / 5 = 0.09

References:

1) Humanic. (2013). www.physics.ohio-state.edu/~humanic/. In Thomas Humanic Brochure Page.

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The edition was dedicated to the memory of Stella Kupferberg, Director of the Photo Department: "We miss you, Stella, and shall always remember that a well-chosen photograph should speak for itself, without the need for a lengthy explanation"

3) Martindale, D. G. & Heath, R. W. & Konrad, W. W. & Macnaughton, R. R. & Carle, M. A. (1992). *Heath Physics.* Lexington: D.C. Heath and Company

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