## Relative Motion

## by

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P2.3a: Describe and compare the motion of an object using different reference frames. Relative Motion: PI.2f: Critique solutions to problems, given criteria and scientific constraints.

## Items:

1. Vector, Scalar Quantities.
2. Adding Vector Quantities in the Same or Opposite Directions.
3. Relative Motion.

## Vector, Scalar

A scalar quantity is one that can be described by a single number: temperature, speed, mass

A vector quantity deals inherently with both magnitude and direction: velocity, force, displacement

Arrows are used to represent vectors. The direction of the arrow gives the direction of the vector.

By convention, the length of a vector arrow is proportional to the magnitude of the vector. 8 is the double of 4 . The length of vector 8 lb is twice that of 4 lb .

4 lb


## Adding Vectors

Often it is necessary to add one vector to another. Vectors are added by joining them head to tail. The vector sum is represented by an arrow drawn from the tail of the first vector to the head of the last vector.

$A$ and $B$ are two vectors in the same direction. $R$ is the sum or resultant vector. Notice how $A$ and $B$ are joined tail to head to give the total length of $R$.

## Rules for Adding Vectors

To add two vectors on the same direction, combine them by adding their magnitudes together.

To add two vectors that are in opposite directions, change the sign of one vector so that they have the same direction, and add their magnitudes.

Example 1. Displacement vectors in one dimension: In the figure below, a man first moves along a straight line with a displacement vector A of 5 meters in the east direction. Then, the man moves again in the same direction with a displacement $B$ of 3 meters. These two vectors ass to give the total displacement vector $R$, which would apply if the man moves from start to finish in one step. The addition is written as: $R=A+B=5+3=8 \mathrm{~m}$.


8 m

## Application: Relative Motion

The figure below shows an experiment performed inside a uniformly moving train.


When the keys are tossed straight up. As seen from inside, the tossed keys go straight up. As seen from the outside, the keys appear to move along an ascending parabola (shown as the orange line path).

A reference frame describe the orientation of an object's motion with regard to its position, distance, speed, and direction.

When an object is in motion in a reference frame, that is also in motion, the motion of the object will generally appear different to an observer at rest outside the reference frame versus an observer at rest in the reference frame.

## Example 2:

Suppose a train is moving at $20 \mathrm{~km} / \mathrm{h}$ [E]. Seated passengers are not moving relative to the train and for someone who is sitting in the train. For someone who is standing outside, passengers are moving at $20 \mathrm{~km} / \mathrm{h}$ [E] relative to the ground.

Likewise, a passenger walking inside a moving train has different velocities relative to the train and the ground.

## Adding Velocity Vectors in one Direction

To add two vectors in the same direction, combine them by adding their magnitude together.

| Relative Velocity |
| :---: |
| ${ }_{1} \overrightarrow{V_{3}}={ }_{1} \overrightarrow{V_{2}}+{ }_{2} \vec{V}_{3}$ |

${ }_{1} \vec{V}_{3},{ }_{1} \vec{V}_{2},{ }_{2} \vec{V}_{3}$ are vector quantities representing velocities.
${ }_{1} \vec{V}_{2}, 1$ represent the head of the arrow vector $\overrightarrow{\mathrm{V}}$, and 2 represent the tail of the arrow vector
${ }_{2} \vec{V}_{3}, 2$ represent the head of the arrow vector $\overrightarrow{\mathrm{V}}$, and 3 represent the tail of the arrow vector
${ }_{1} \overrightarrow{V_{3}}, 1$ represent the head of the arrow vector $\overrightarrow{\mathrm{V}}$, and 3 represent the tail of the arrow vector

## ${ }_{1} \vec{V}_{3}={ }_{1} \vec{V}_{2}+{ }_{2} \vec{V}_{3}$



Suppose that:
${ }_{1} V_{2}=30 \mathrm{~km} / \mathrm{h}[\mathrm{E}]$. This is a velocity (speed) in the east direction.
${ }_{2} V_{3}=20 \mathrm{~km} / \mathrm{h}[\mathrm{E}]$. This is another velocity (speed) in the east direction.
The sum is:
${ }_{1} V_{3}={ }_{1} \vec{V}_{2}+{ }_{2} \vec{V}_{3}=30 \mathrm{~km} / \mathrm{h}[\mathrm{E}]+20 \mathrm{~km} / \mathrm{h}[\mathrm{E}]=50 \mathrm{~km} / \mathrm{h}[\mathrm{E}]$

## Example 3:


$\mathbf{v} \mathbf{P T}=$ velocity of the Passenger relative to the Train $=+2.0 \mathrm{~m} / \mathrm{s}$
$\mathbf{v} \mathbf{T G}=$ velocity of the Train relative to the Ground $=+9.0 \mathrm{~m} / \mathrm{s}$
$\mathbf{v} \mathbf{P G}=$ velocity of the Passenger relative to the Ground $=+11 \mathrm{~m} / \mathrm{s}$

$$
v_{P G}=v_{P T}+v_{T G}
$$

$$
\mathbf{v}_{\mathbf{P G}}=(2.0 \mathrm{~m} / \mathrm{s})+(9.0 \mathrm{~m} / \mathrm{s})=+11 \mathrm{~m} / \mathrm{s}
$$

Example 4: A passenger in a moving train: A train is moving at $20 \mathrm{~km} / \mathrm{h}[\mathrm{E}] . \mathrm{A}$ passenger is walking inside a moving train.
i) If a passenger on this train walks towards the front of the train at speed of 3 $\mathrm{km} / \mathrm{h}$. What is the passenger's velocity (speed) relative to the ground (or for someone standing outside)?

The velocity of a passenger relative to the ground $(\mathrm{p} V \mathrm{~g})=$ passenger velocity relative to the train $(\mathrm{p} V \mathrm{t})+$ the train's velocity relative to the ground $(\mathrm{t} V \mathrm{~g})$.

$$
\mathrm{p} \vec{V} \mathrm{~g}=\mathrm{p} \vec{V} \mathrm{t}+\mathrm{t} \vec{V} \mathrm{~g}
$$

V: velocity, p: passenger, g: ground, t : train
$\mathrm{p} \overrightarrow{\mathrm{V}} \mathrm{g}=$ velocity of the passenger relative to the ground ?
$\overrightarrow{\mathrm{p} V} \mathrm{t}=$ velocity of the passenger relative to the train $=3 \mathrm{~km} / \mathrm{h}[\mathrm{E}]$
$\overrightarrow{\mathrm{t} V} \mathrm{~g}=$ velocity of the train relative to the ground $=20 \mathrm{~km} / \mathrm{h}[\mathrm{E}]$
$\overrightarrow{\mathrm{t} V} \mathrm{~g}=20 \mathrm{~km} / \mathrm{h}[\mathrm{E}]$

$\overrightarrow{\mathrm{p} V} \mathrm{t}=3 \mathrm{~km} / \mathrm{h}[\mathrm{E}]$


The passenger and the train are going in the same direction (east, or positive), then the vectors are added.
$\mathrm{p} \overrightarrow{V \mathrm{~g}}=\mathrm{p} \overrightarrow{\mathrm{V}}+\mathrm{t} \overrightarrow{V \mathrm{~g}}=3+20=23 \mathrm{~km} / \mathrm{h}[\mathrm{E}]$.


So the passenger velocity relative to the ground is $23 \mathrm{~km} / \mathrm{h}$ [ $E$ ].
ii) If the passenger walks towards the back of the train with a velocity of $3 \mathrm{~km} / \mathrm{h}$ $[\mathrm{W}]$. What is the relative velocity of the passenger relative to the ground?

Remember: To add two vectors that are in opposite directions, change the sign of one vector so that they have the same direction, and add their magnitude.
$\overrightarrow{\mathrm{t} V} \mathrm{~g}=20 \mathrm{~km} / \mathrm{h}[$ East $]$
t g
$\overrightarrow{\mathrm{p} V} \mathrm{t}=3 \mathrm{~km} / \mathrm{h}$ [West]
Change the sign

$\rightarrow$
$\mathrm{p} V \mathrm{t}=-3 \mathrm{~km} / \mathrm{h}$ [East]
$\mathrm{p} \overrightarrow{\mathrm{g}}=\mathrm{p} \vec{V} \mathrm{t}+\mathrm{t} \vec{V} \mathrm{~g}$

$$
\begin{aligned}
& =3 \mathrm{~km} / \mathrm{h}[\mathrm{~W}]+20 \mathrm{~km} / \mathrm{h}[\mathrm{E}] \\
& =20 \mathrm{~km} / \mathrm{h}[\mathrm{E}]-3 \mathrm{~km} / \mathrm{h}[\mathrm{E}] \\
& =17 \mathrm{~km} / \mathrm{h}[\mathrm{E}]
\end{aligned}
$$



## For Airplane Navigation Problems:

The airplane velocity relative to the ground is found by adding the plane's velocity relative to the air and the velocity to the air relative to the ground.

$$
\mathrm{p} \vec{V} \mathrm{~g}=\mathrm{p} \vec{V} \mathrm{a}+\mathrm{a} \vec{V} \mathrm{~g}
$$

V; velocity, p; plane, g; ground a; air
$\mathrm{p} \vec{V} \mathrm{~g}=\mathrm{p} \vec{V} \mathrm{a}+\overrightarrow{\mathrm{a} V} \mathrm{~g}$
$\mathrm{p} \overrightarrow{\mathrm{V}} \mathrm{g}=$ Plane's velocity relative to the ground.
$\mathrm{p} \overrightarrow{\mathrm{V}} \mathrm{a}=$ Plane's velocity relative to the air.
$\overrightarrow{\mathrm{a}} \overrightarrow{\mathrm{g}}=$ Wind velocity (velocity of air relative to the ground)

## Example 5:



Suppose that there is an airplane traveling at $80 \mathrm{~km} / \mathrm{h}$ relative to the ground below in the north direction. Suppose, also, that there is a tailwind traveling at 20 $\mathrm{km} / \mathrm{h}$ north. With the tailwind, the airplane travels at $80+20=100 \mathrm{~km} / \mathrm{h}$ north.

Now, suppose that the wind change direction to be $20 \mathrm{~km} / \mathrm{h}$ south. So, the airplane is now traveling 80-20 $=60 \mathrm{~km} / \mathrm{h}$ north.

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The edition was dedicated to the memory of Stella Kupferberg, Director of the Photo
Department: "We miss you, Stella, and shall always remember that a well-chosen photograph should speak for itself, without the need for a lengthy explanation"
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