Graphical Velocity and Acceleration

by

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P2.1D Describe and analyze the motion that a position-time graph represents, given the graph.

P2.2C Describe and analyze the motion that a velocity-time graph represents, given the graph.

P2.2e Use the area under a velocity-time graph to calculate the distance traveled and the slope to calculate the acceleration.

Items:

1. Velocity: Slope of Position versus Time Graph.

2. Instantaneous Velocity.

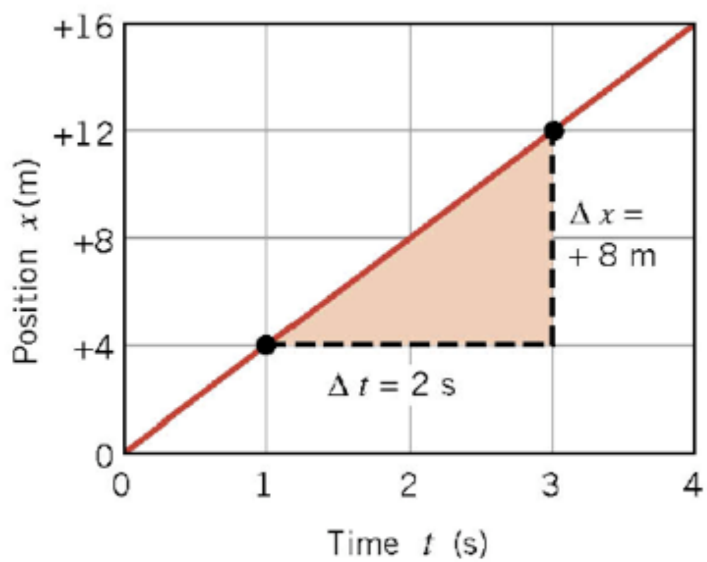
3. Acceleration: Slope of Velocity versus Time Graph.

Velocity (symbole: v)

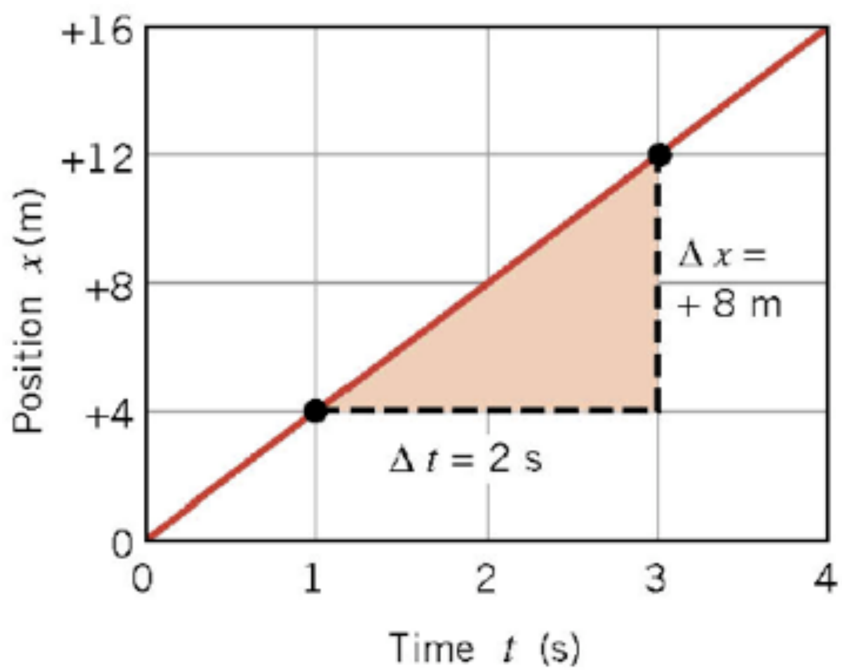
Constant Velocity: Slope of Position versus Time Graph

Suppose a bicyclist is riding with a constant velocity of v = 4 m/s. The position x of the bicycle can be plotted along the vertical axis of a graph. The time t is plotted along the horizontal axis. A graph of position versus time is shown below.

Recall that linear equations have the general form: $y = mx$ (m is a constant)



Since the position of the bike increases by 4 m every second, the graph of x versus t is a straight line. Furthermore, if the bike is assumed to be at x = 0 m when t = 0 s, the straight line passes through the origin. Each point on this line gives the position of the bike at a particular time. For instance, at t = 1s the position is 4 m, while at t = 3 s the position is 12 m.



The velocity could be determined by considering what happens to the bike between the times of 1s and 3 s, for instance. It is the slope of the line.

Calculation:

Between the time, 1s (initial) and 3 s (final), the change in time is $\Delta t = (t_{final} - t_{initial}) = (3-1) = 2$ s.

During this time interval, the position of the bike changes from **+** 4 m (initial) to **+** 12 m (final). So, the change of position Δx = final position - initial position = $(x_{final} - x_{initial}) = (12 - 4) = 8$ m.

The ratio $\Delta x / \Delta t$ is called the **slope** of the straight line and it is the velocity.

$$\text{Velocity} = \text{Slope} = \Delta x / \Delta t$$

$$\text{Slope} = \frac{\Delta x}{\Delta t} = \frac{+8\text{ m}}{2\text{ s}} = +4\text{ m/s} = v$$

Calculating Velocity from a position-time graph.

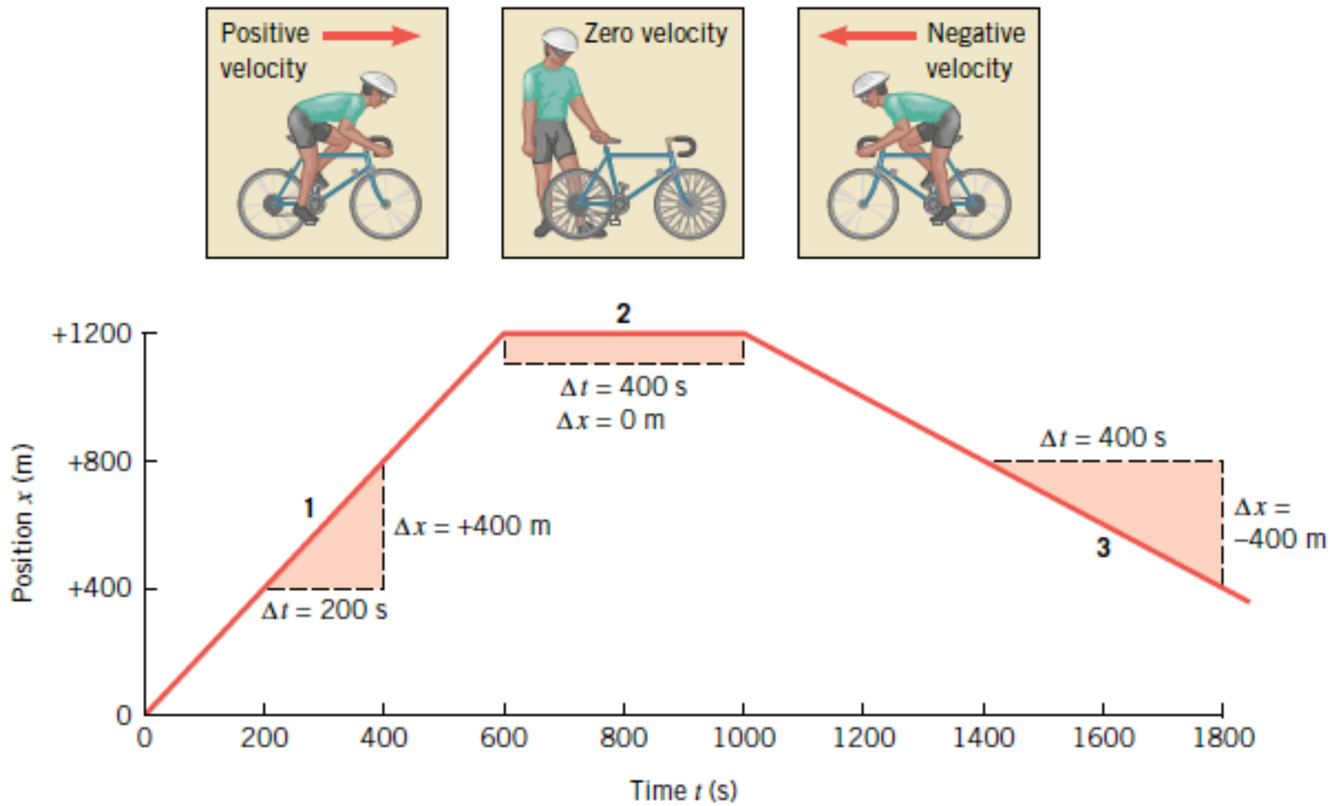
- Thus, for an object moving with a constant velocity, the slope of the straight line in a position– time graph gives the velocity. The position (x) is plotted along the vertical axis of a graph. The time (t) is plotted along the horizontal axis.

- Since the position– time graph is a straight line, any time interval t can be chosen to calculate the velocity. Choosing a different t will yield a different x, but the velocity $\Delta x / \Delta t$ will not change.

Calculating velocity using a position-time graph
Velocity (v) = slope (x *vs* t)= $\Delta x / \Delta t$ = **change in position / change in time =** $(x_{final} - x_{initial}) / (t_{final} - t_{initial})$

Example 1: *A Bicyclist Riding with a Constant Velocity*

In the real world, objects rarely move with a constant velocity at all times, as illustrated in the position versus time graph of a bicycle trip shown below.



Analysis of the Graph

Segment 1: Positive velocity: A bicyclist maintains a constant positive velocity on the outgoing leg of a trip. When time increases from 0 s to 600 s, the distance increases from 0 m to 1200 m.

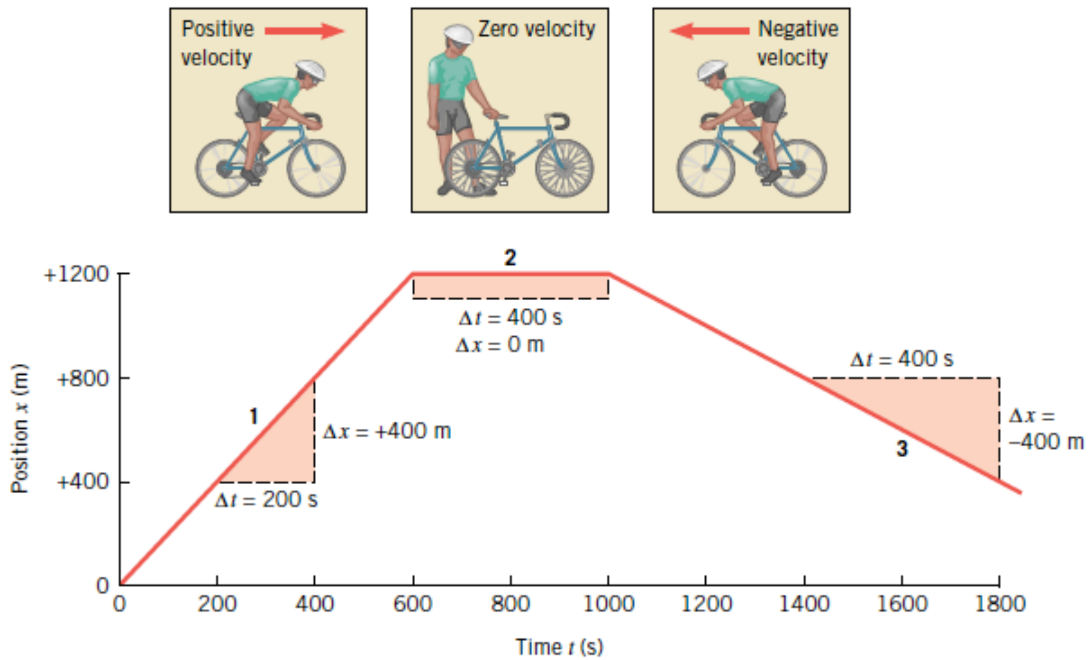
Segment 2: Zero velocity. He maintains zero velocity while stopped. When time increases from 600 s to 1000 s, the distance stays the same at 1200 m (no movement).

Segment 3: Negative velocity. Another constant velocity on the way back. When time increases after 1000 s, the distance decreases from 1200 m and less.

Using the time and position intervals indicated in the drawing, obtain the velocities for each segment (1, 2 and 3) of the trip.

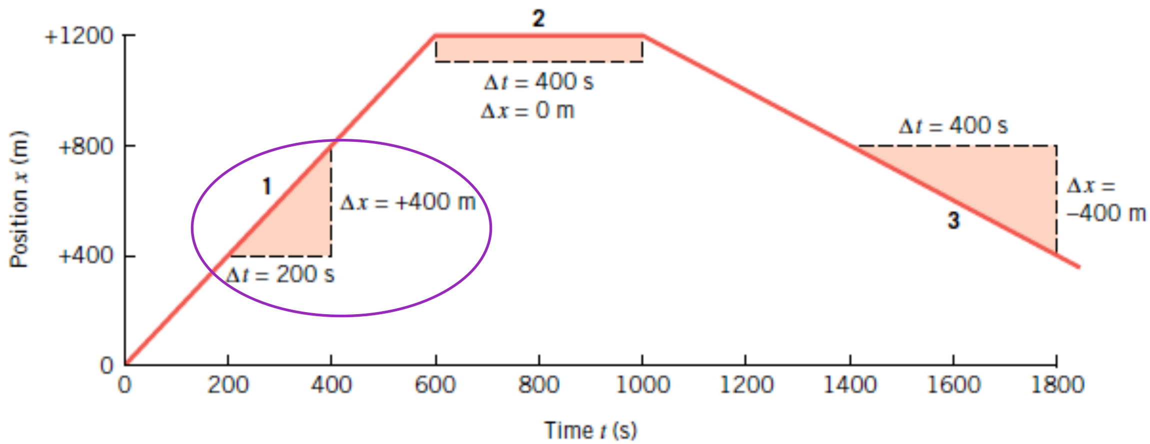
Answer

We know that velocity is the slope to the position-time graph. We need to calculate the slope of each segment. velocity = slope = $\Delta x / \Delta t$.



We can take any two points on each segment and find the slope. Usually, we choose the clear points that make the calculation easier.

Segment 1:



During the time between 200 s ($t_{initial}$) and 400 s (t_{final}), the distance went from 400 m ($x_{initial}$) to 800 m (x_{final}).

Velocity = slope = change in position / change in time = Δx /Δt.

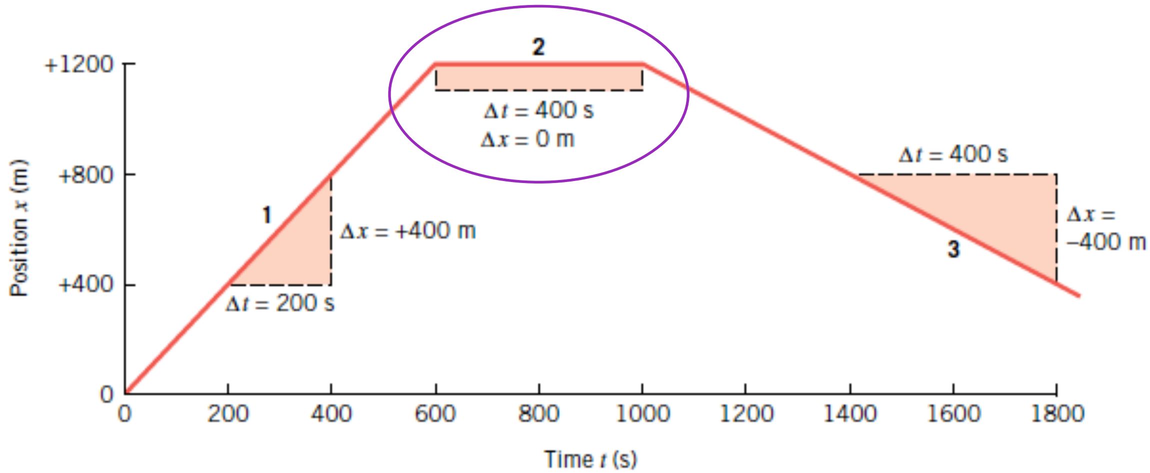
$$(x_{final} - x_{initial}) / (t_{final} - t_{initial}) =$$

(800-400) / (400-200) =

400/200 =

2 m/s

Segment 2:



During the time from 600 s ($t_{initial}$) to 1000 s (t_{final}), the distance stayed the same at 1200 m.

Velocity = slope = change in position / change in time = Δx /Δt.

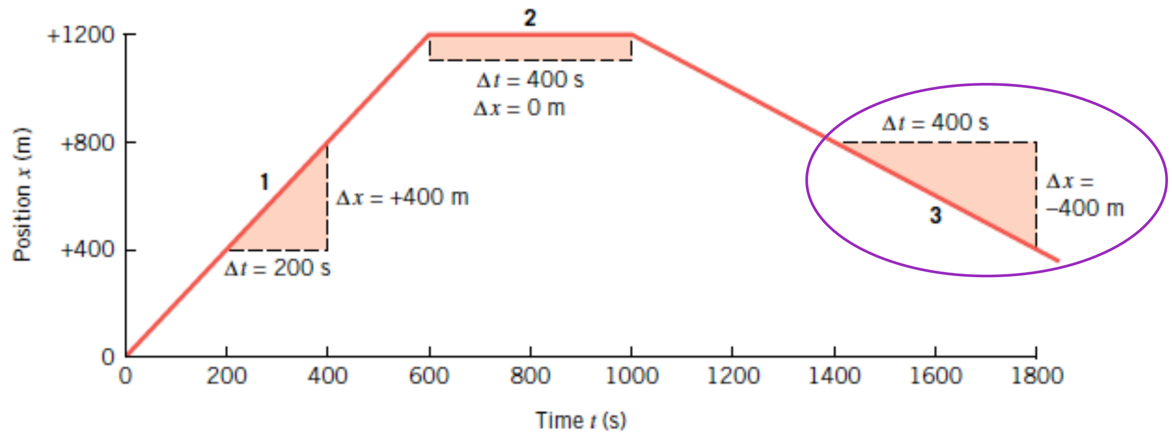
$(x_{final} - x_{initial}) / (t_{final} - t_{initial}) =$

(1200-1200) / (1000-600) =

0/400 =

0 m/s (not moving)

Segment 3:



During the time from 1400 s ($t_{initial}$) to 1800 s (t_{final}), the distance changed from 800 m ($x_{initial}$) to 400 m (x_{final}). He is going back toward initial position.

Velocity = slope = change in position / change in time = Δx /Δt.

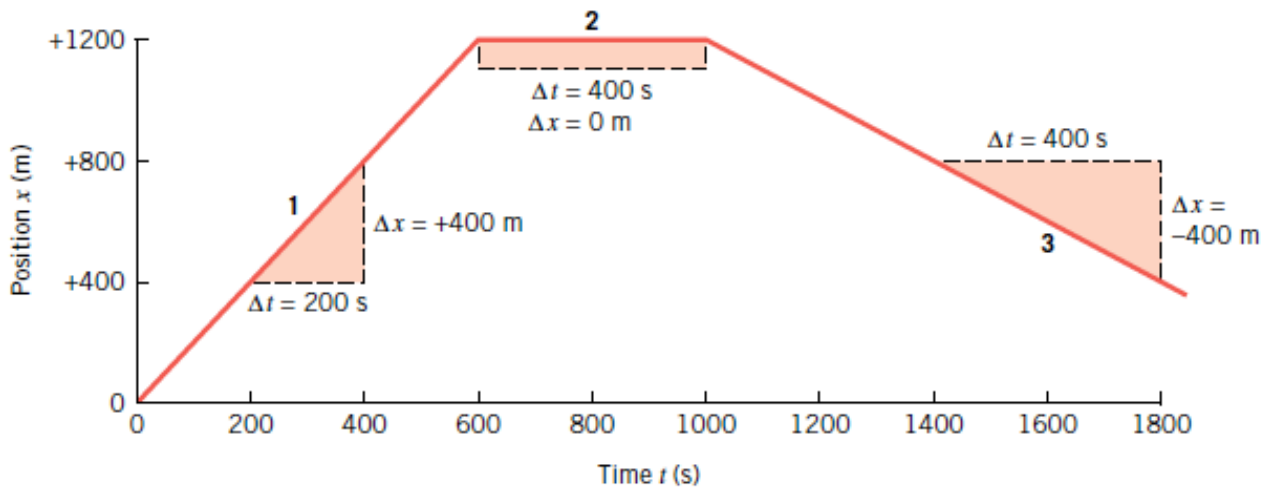
$(x_{final} - x_{initial}) / (t_{final} - t_{initial}) =$

(400- 800) / (1800-1400) =

-400/ 400 =

-1 m/s (negative velocity)

Summary



Solution The average velocities for the three segments are

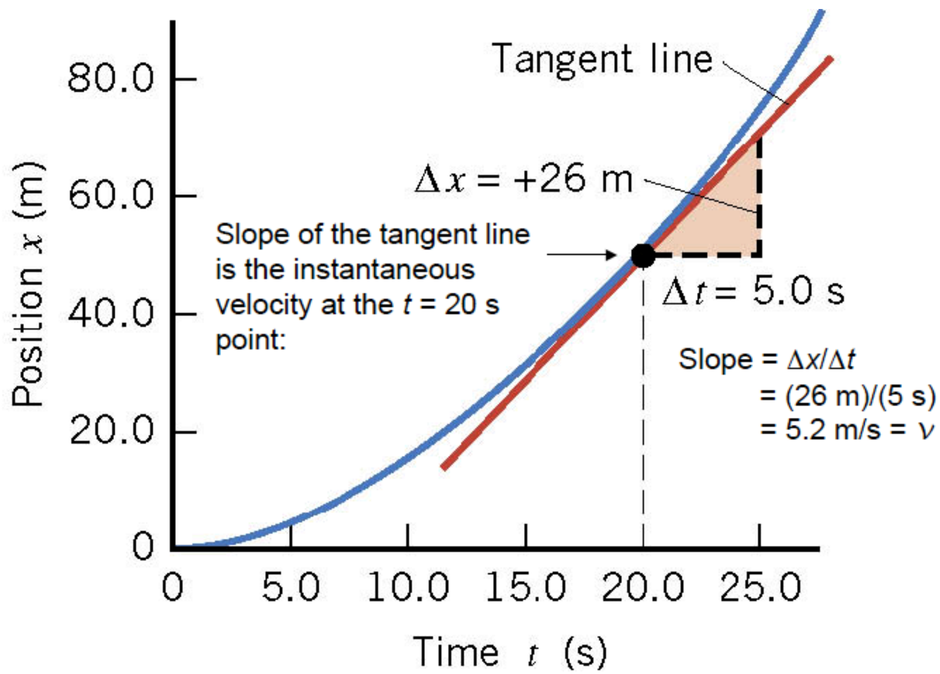
Segment 1 $$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{800\text{ m} - 400\text{ m}}{400\text{ s} - 200\text{ s}} = \frac{+400\text{ m}}{200\text{ s}} = \boxed{+2\text{ m/s}}$$

Segment 2 $$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{1200\text{ m} - 1200\text{ m}}{1000\text{ s} - 600\text{ s}} = \frac{0\text{ m}}{400\text{ s}} = \boxed{0\text{ m/s}}$$

Segment 3 $$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{400\text{ m} - 800\text{ m}}{1800\text{ s} - 1400\text{ s}} = \frac{-400\text{ m}}{400\text{ s}} = \boxed{-1\text{ m/s}}$$

Instantaneous Velocity

Object Moving with Changing Velocity

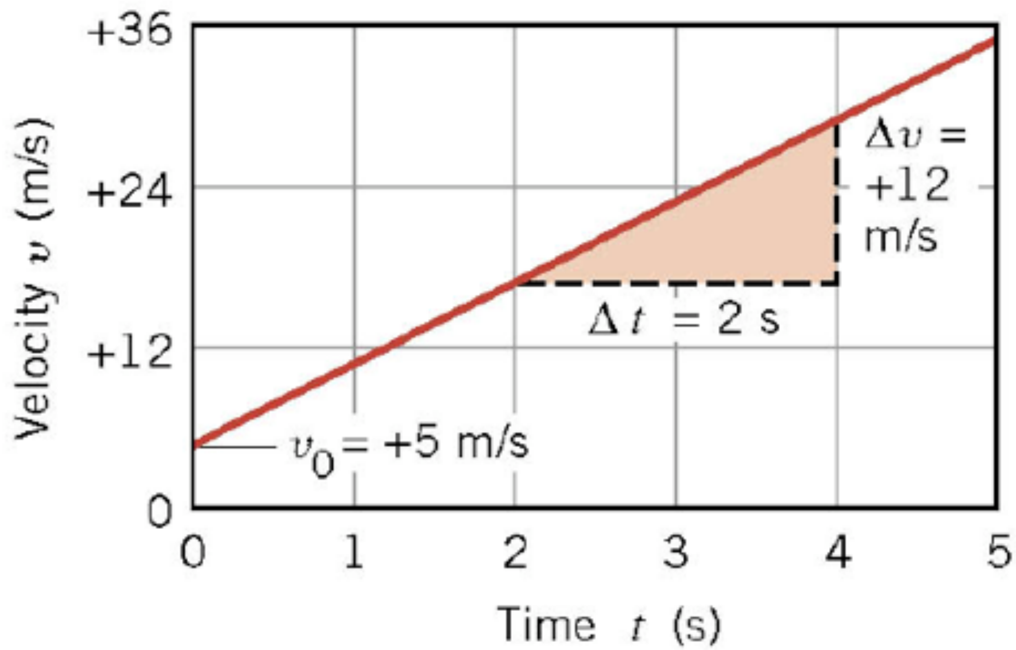


When the velocity is changing, the position-vs.-time graph is a curved line. The slope **Δx/Δt** of the tangent line drawn to the curve at a given time is the instantaneous velocity at that time.

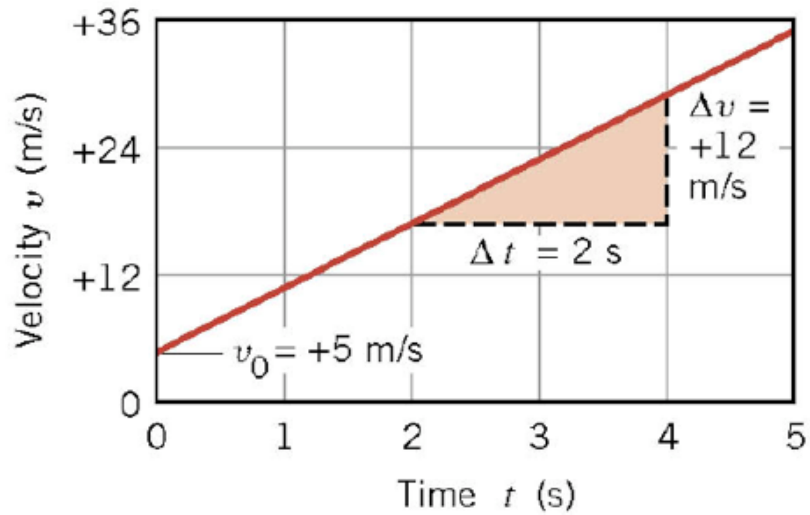
Acceleration (symbol: a)

Acceleration: Slope of Velocity versus Time Graph

- For an object moving with a constant velocity, the slope of the straight line in a velocity– time graph gives the acceleration. The velocity (v) is plotted along the vertical axis of a graph. The time (t) is plotted along the horizontal axis.



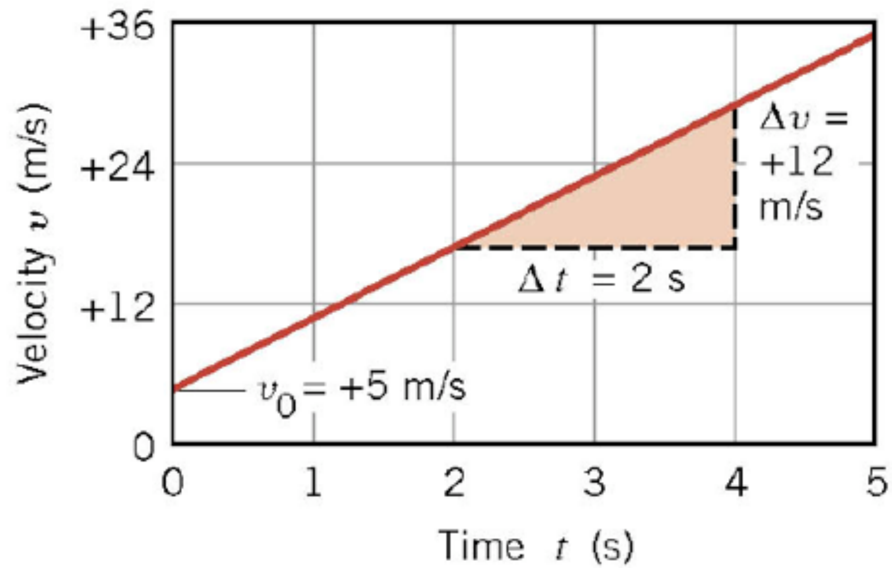
• Since the velocity– time graph is a straight line, any time interval t can be chosen to calculate the acceleration. Choosing a different t will yield a different v, but the velocity **Δ**v /**Δ**t will not change.



Calculating acceleration using a velocity-time graph
Acceleration (a) = slope (v *vs* t) = **Δ**v /**Δ**t = **change in velocity / change in time =** $(v_{final} - v_{initial}) / (t_{final} - t_{initial})$

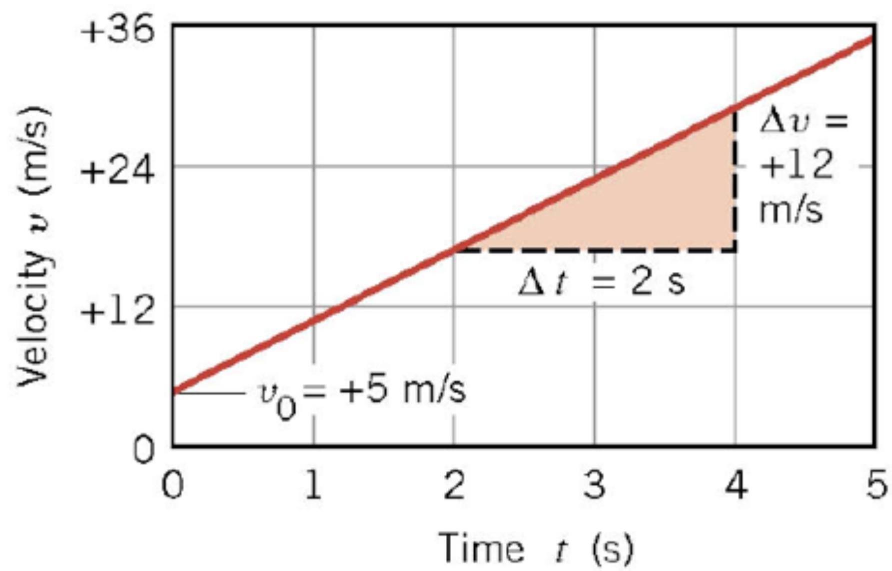
Example 2: *An object moving at constant acceleration.*

This is the velocity versus time graph of an object moving at constant acceleration. The graph is a straight line (red color). The initial velocity is $\mathbf{v}_0$ = 5 m/s when t **=** 0 s. Calculate the acceleration of this object.



To calculate the acceleration, we need to calculate the slope of the line. We can take the time interval between 2 s (initial) and 4 s (final). At t = 2 s, the velocity (v_{initial}) is 16 m/s. At t = 4 s, the velocity (v_{final}) is 28 m/s.

Calculation:



Acceleration = slope = change in velocity / change in time =

$\Delta v / \Delta t =$

$(v_{final} - v_{initial}) / (t_{final} - t_{initial}) =$

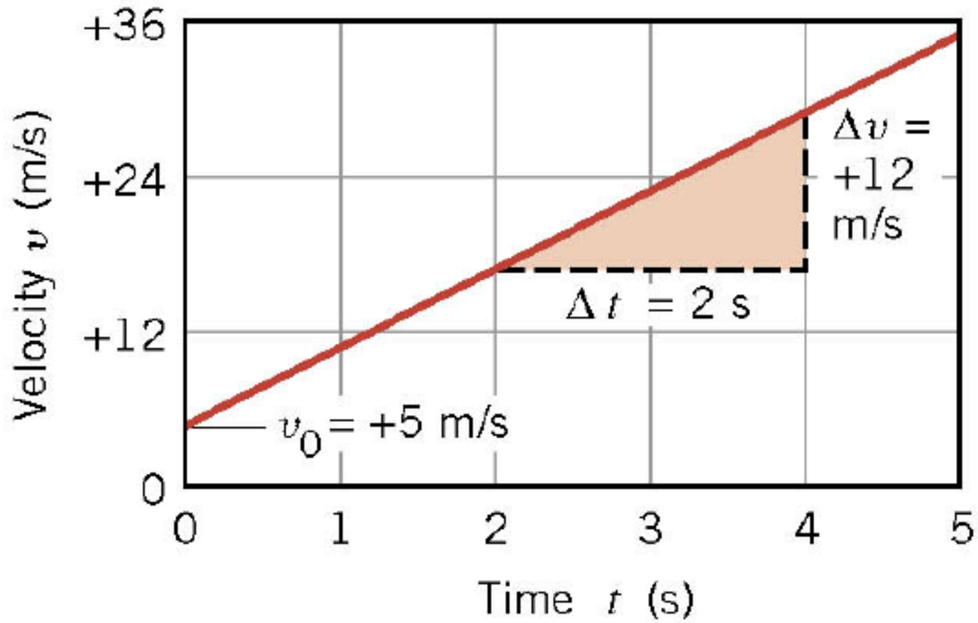
(28 - 16)/ (4 - 2)=

12 / 2 =

6 m/s^2

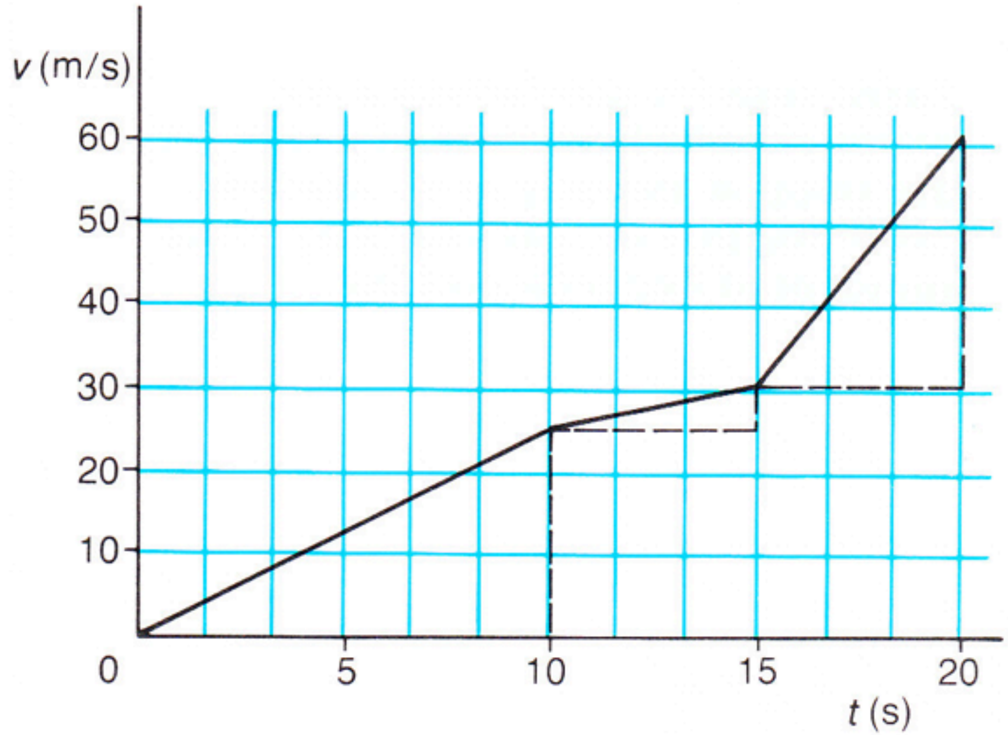
Summary:

This velocity-vs.-time graph applies to an object with an acceleration of **Δv**/**Δ**t **=** 6 m/s^2.



$$\text{Slope} = \frac{\Delta v}{\Delta t} = \frac{+12\ \text{m/s}}{2\ \text{s}} = +6\ \text{m/s}^2 \quad = a$$

Example 3:



References:

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2) Cutnell, J. D. & Johnson, K. W. (1998). *Cutnell & Johnson Physics, Fourth Edition*. New York: John Wiley & Sons, Inc.

The edition was dedicated to the memory of Stella Kupferberg, Director of the Photo Department: "We miss you, Stella, and shall always remember that a well-chosen photograph should speak for itself, without the need for a lengthy explanation"

3) Martindale, D. G. & Heath, R. W. & Konrad, W. W. & Macnaughton, R. R. & Carle, M. A. (1992). *Heath Physics.* Lexington: D.C. Heath and Company

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6) Nada H. Saab (Saab-Ismail), (2009- 2014) Wayne RESA, Bilingual Department.